A Novel Robust Optimization Method for Fleet Sizing and Differentiated Pricing in Bike-Sharing Systems

Haiyu Kan, Chengqing Yu, Ruiyou Zhang, and Peng Liu

***Abstract*—This study proposes an integrated fleet sizing and dynamic pricing (IFSDP) problem for bike-sharing systems to alleviate the imbalanced distribution of bikes during operations. In the IFSDP problem, the number of bikes deployed at each area in the system and the differentiated trip prices for different origin-destination pairs in different periods are jointly determined, taking into account demand uncertainty. A bi-level robust mixed-integer programming model is developed to characterize the interactions between the bike-sharing platform and the uncertain operating environment. The model is reformulated into a tractable mixed-integer quadratic programming model and further relaxed to achieve a trade-off between solution robustness and objective value loss. The proposed methods are validated and evaluated using extensive numerical experiments. Results demonstrate that the reformulated model is applicable in practice in terms of computational efficiency, and that the decisions can help increase both the profit and the number of served users, compared to the deterministic benchmark. The superiority of the proposed methods is independent of the type of uncertainty.**

***Key Words*—bike-sharing system; fleet sizing; differentiated pricing; improved robust optimization**

I. INTRODUCTION

BIKE -sharing services address the “last-mile” problem in urban mobility, and have attracted widespread attention worldwide in recent years. With the continuous diffusion of the services, the imbalanced distribution of bikes among areas has gradually become the main operational challenge for the platform [1]. Due to the tidal and uncertain nature of the demand, some areas may accumulate excessive bikes and cause difficulties for users to return bikes, while other areas may suffer from insufficient bike availability and fail to meet user demand. The imbalanced bike distribution not only reduces users’ satisfaction but also weakens the road capacity and increases traffic safety risks. Therefore, efficient bike relocations are urgent for the long-term development of the bike-sharing industry [2].

Shared-bike relocations can be roughly categorized into three approaches: system configuration optimization, user-based relocations, and operator-based relocations. System configuration optimization aims at coordinating supply and demand statically by optimizing the service area or the bike deployment before the operations [3]. User-based and operator-based relocations focus on balancing supply anddemand dynamically through relocation activity executions during the operations [4]. For user-based relocations, the platform applies dynamic or differentiated trip prices to adjust the demand quantities and patterns for areas, thereby aligning demand better with supply [5, 6]. For operator-based relocations, the platform employs dedicated staff to carry bikes from over-supplied areas to under-supplied areas using relocating trucks [7].

In the field of user-based relocations, He et al. [8] induced users to relocate shared bikes from areas with sufficient supply to those with insufficient supply through price incentives. Zhang et al. [9] established a bi-level mixed-integer programming model to decide the incentives offered to users relocating vehicles, where users’ choices under incentives were explicitly incorporated in the lower level. Since users exhibit heterogeneous and uncertain price sensitivities to incentives and price variations, an accurate formulation of user behavior under incentives or price variations is crucial for optimizing user-based relocations. An online learning method was developed in [10] to update and estimate users’ sensitivity based on the previous users’ behaviors under incentives. Jin et al. [11] described users’ selections towards renting and returning stations under incentives as opportunity constraints.

Operator-based relocations are similar to pickup and delivery problems, where the routes of relocating trucks and the quantities of bikes to be loaded or unloaded at each area should be determined. Lv et al. [12] designed a multi-objective evolutionary algorithm to minimize the fuel consumption costs of relocations and the unmet relocation tasks. Gleditsch et al. [13] investigated a dynamic operator-based relocation problem using a rolling-horizon framework. A column generation heuristic was designed to obtain solutions within a few seconds

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H. Kan is with the School of Management, Shenyang University of Technology, Shenyang 110870, China (e-mail: kanhaiyu@sut.edu.cn).

C. Yu is with the School of Management, Shenyang University of Technology, Shenyang 110870, China (e-mail: 15040457029@163.com).

R. Zhang is with College of Information Science and Engineering, Northeastern University, Shenyang 110819, China (e-mail: zhangruiyou@ise.neu.edu.cn).

P. Liu is with the School of Management, Shenyang University of Technology, Shenyang 110870, China (e-mail: liup7802@163.com).

Corresponding author: Haiyu Kan.

in real-time. Cui et al. [14] reported a mixed relocation problem with both man-powered shared bikes and e-bikes. A hybrid genetic search algorithm was developed to determine the activities of relocating trucks and the recharging of e-bikes at stations.

System configuration optimization is typically studied in conjunction with user-based or operator-based relocation strategies, since relocation activities directly influence the efficiency of resource allocation. For example, Datner et al. [15] considered users' spontaneous adjustment of stations when bikes are not available at their desired stations, and designed a simulation-based optimization algorithm to determine the bike deployment at stations. Fu et al. [16] established a two-stage optimization model for the integrated optimization of system configuration and operator-based relocations, with the system configuration and operator-based relocations during the planning horizon determined in the first and second stages, respectively. Jiang and Ouyang [17] considered the competition among multiple bike-sharing platforms. Strategic and operational decisions, in terms of station locations, fleet sizes, pricing, and operator-based relocations, for different platforms are determined simultaneously in a game-theoretical model.

Some researchers considered the uncertainty in demand and adopted stochastic approaches to optimize the system configuration or operator-based relocations. Wu and Li [18] formulated the uncertain demand as a scenario tree, and established a multi-stage stochastic programming model to determine the number of relocating trucks required to complete the given relocation tasks. Zhou et al. [19] considered battery swapping and bike relocation simultaneously for electric bike-sharing systems, and divided the dedicated staff into the battery-swapping group and the relocating group for completing the two types of tasks. Markov decision theory was developed to optimize the task execution of the two groups while considering the locations of battery cabinets and the number of available batteries in the cabinets. Chen et al. [20] focused on bike relocations in the free-floating systems, where bike collections in the areas should be conducted before relocations. An improved distributionally robust method was developed to optimize the bike collections and relocations conducted by dedicated staff.

To the best of our knowledge, differentiated or dynamic pricing for user-based relocations under uncertain demand has seldom been reported. In addition, robust optimization, as an effective method for addressing uncertainty, avoids reliance on probability distributions of unknown parameters and has been widely applied in fields such as logistics and transportation planning [21]. However, robust optimization has not been adopted to optimize user-based relocations in bike-sharing [20]. The main reason is that the demand for shared bikes varies with the relocation parameters in user-based relocation schemes, while the demand itself exhibits uncertainty. Formulating the uncertain and varied demand poses a significant challenge in establishing and solving the optimization models.

To enrich the field of bike-sharing relocations, differentiated pricing under uncertain demand and price sensitivity is investigated in this research under the framework of robust optimization. The bike deployment and the differentiated prices are optimized simultaneously at the tactical level to enhance the relocation efficiency. The contributions of this research are summarized as follows.

(i) An integrated fleet sizing and differentiated pricing problem (called IFSDP problem) is proposed for bike-sharing to address the imbalances of distribution of available bikes, where the demand uncertainty is considered explicitly. A bi-level mixed-integer programming model is formally formulated the problem.

(ii) The bi-level mixed-integer programming model is reformulated into a mixed-integer quadratic model based on the strong duality theorem. The reformulated model is further improved through constraint relaxations to trade off the solution robustness against the loss in the objective value.

The remainder of this paper is structured as follows. Section II presents the problem description of the IFSDP problem, followed by the mathematical formulation in Section III. Section IV reformulates the bi-level model into a tractable one and introduces the constraint-relaxation strategy. The proposed methods are validated and evaluated in Section V, with the conclusions and further possible research in Section VI.

II. IFSDP PROBLEM

Consider a bike-sharing platform that provides shared bike services within a certain region. The service region is divided into multiple areas based on population densities and community functions. The imbalanced distribution of bikes occurs regularly during the operations due to the tidal nature of travel demand. To alleviate the imbalances, the platform matches the supply with demand by coordinating the bike deployment with dynamic and differentiated trip prices. The planning horizon is divided into multiple periods based on the pattern of the potential demand.

Differentiated trip prices are applied among different origin-destination (O-D) pairs in each period to adjust the demand pattern. Intuitively, as the trip price increases, the number of users willing to use shared bikes decreases, leading to a decreasing function between the trip price and demand. Demand uncertainty is also taken into consideration. The variation relationship between the actual volume of demand $\widetilde{d}_{ij}^t$ and the corresponding trip price of using a shared bike p_{ij}^t is assumed as follows:

$$\widetilde{d}_{ij}^t = \widetilde{a}_{ij}^t - \widetilde{b}_{ij}^t p_{ij}^t \tag{1}$$

where $\widetilde{a}_{ij}^t \in [a_{ij}^t - \hat{a}_{ij}^t,\ a_{ij}^t + \hat{a}_{ij}^t]$ denotes the actual maximum base of users. The actual maximum user base is attained when the trip price is zero. a_{ij}^t is the estimated maximum base of users; $\hat{a}_{ij}^t (> 0)$ is the fluctuation volume for the maximum base of users. $\widetilde{b}_{ij}^t \in [b_{ij}^t - \hat{b}_{ij}^t,\ b_{ij}^t + \hat{b}_{ij}^t]$ is the actual price elasticity of demand, which reflects users' price sensitivity. b_{ij}^t is the estimated price elasticity of demand. $\hat{b}_{ij}^t$ (> 0) is the fluctuation volume for the price elasticity of demand.

The IFSDP problem determines the number of shared bikes deployed at each area at the beginning of the planning horizon and the differentiated trip prices for O-D pairs at each period, so that the operating profit over the planning horizon can be maximized. The operating profit is the revenue from riding minus the depreciation cost of the deployed bikes. We assume that all trips can be completed within a single period, i.e., a rented bike can be returned in the corresponding period, given the long duration of a period. Such an assumption is common in the literature focusing on bike-sharing relocations, such as in [17] and [22]. The sets and parameters in the problem are summarized in Table I.

TABLE I Sets and Parameters in the IFSDP Problem

Notation	Meaning
I	Set of areas
T	Set of periods, $T = \{1, 2, \cdots, \lvert T\rvert\}$
$\tilde{a}_{ij}^t$	Actual maximum base of users driving from area $i \in I$ to area $j \in I$ in period t
a_{ij}^t	Estimated maximum base of users driving from area $i \in I$ to area $j \in I$ in period $t \in T$
$\hat{a}_{ij}^t$	Fluctuation volume for the maximum base of users driving from area $i \in I$ to area $j \in I$ in period $t \in T$
$\tilde{b}_{ij}^t$	Actual price elasticity of demand driving from area $i \in I$ to area $j \in I$ in period $t \in T$
b_{ij}^t	Estimated price elasticity of demand driving from area $i \in I$ to area $j \in I$ in period $t \in T$
$\hat{b}_{ij}^t$	Fluctuation volume for the price elasticity of demand driv- ing from area $i \in I$ to area $j \in I$ in period $t \in T$
c_0	Depreciation cost of a bike in a planning horizon

III. MATHEMATICAL FORMULATION

Robust optimization is a proper method to deal with the IFSDP problem because the accurate distribution of the coefficients in the Eq. (1) is unknown. This section develops a bi-level robust programming model (called Model BL-RP) for the IFSDP problem. Section III.A defines the variables in the model, followed by the formulations of upper level and lower level in Section III.B and Section III.C, respectively.

A. Variables

Model BL-RP formulates the decisions of the bike-sharing platform and the virtual adversary (representing the uncertain operating environment) in the upper and lower levels, respectively (as in Fig.1). The platform first decides the bike deployment and differentiated trip prices to maximize the operating profit, with the consideration of reactions of the virtual adversary. Observing the decisions of the platform, the virtual adversary chooses the scenarios in the uncertainty sets to resist the objective of the platform as much as possible. The variables of Model BL-RP are defined as follows:

Decision variables for the upper level:

n_i^0: Number of shared bikes available at area $i \in I$ at the beginning of the planning horizon

p_{ij}^t: price for a trip traveling from area $i \in I$ to area $j \in I$ in period $t \in T$

Auxiliary variables for the upper level:

K:Total number of shared bikes deployed in the system

n_i^t: Number of shared bikes available at area $i \in I$ at the beginning of period $t \in T$

Decision variables for the lower level:

x_{ij}^t: variation range of the actual demand $\tilde{a}_{ij}^t$,$x_{ij}^t \in [-1, 1]$

y_{ij}^t:variation range of the actual price elasticity of demand $\tilde{b}_{ij}^t$, $y_{ij}^t \in [-1, 1]$

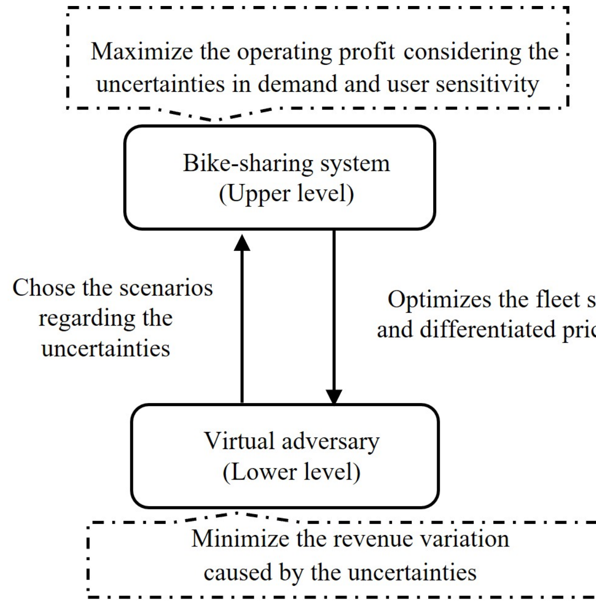


Fig. 1. Decision process in the IFSDP problem

We have $\tilde{a}_{ij}^t = a_{ij}^t + x_{ij}^t \hat{a}_{ij}^t$ and $\tilde{b}_{ij}^t = b_{ij}^t + y_{ij}^t \hat{b}_{ij}^t$ based on the definition of the variables in the lower level. The absolute deviation between the actual demand $\tilde{a}_{ij}^t$ and the estimated demand a_{ij}^t under price p_{ij}^t can be represented as $\left|\hat{a}_{ij}^t x_{ij}^t - \hat{b}_{ij}^t y_{ij}^t p_{ij}^t\right|$.

B. Upper level

The upper level formulates the decision of the platform, which can be expressed as the following mixed-integer nonlinear programming model:

$$\max F = \sum_{t \in T} \sum_{i \in I} \sum_{j \in I} p_{ij}^t \left(a_{ij}^t - b_{ij}^t p_{ij}^t\right) - c_0 K \tag{2}$$

$$K = \sum_{i \in I} n_i^t, \quad \forall t \in T \tag{3}$$

$$\begin{aligned} n_i^{t+1} = n_i^t &- \sum_{j \in I} \left(a_{ij}^t + \hat{a}_{ij}^t x_{ij}^t - b_{ij}^t p_{ij}^t - \hat{b}_{ij}^t y_{ij}^t p_{ij}^t\right) \\ &+ \sum_{j \in I} \left(a_{ji}^t + \hat{a}_{ji}^t x_{ji}^t - b_{ji}^t p_{ji}^t - \hat{b}_{ji}^t y_{ji}^t p_{ji}^t\right) \\ &\forall t \in T \setminus \{\lvert T\rvert\}, \ \forall i \in I \end{aligned} \tag{4}$$

$$n_i^t \geq \sum_{j \in I} \left(a_{ij}^t + \hat{a}_{ij}^t x_{ij}^t - b_{ij}^t p_{ij}^t - \hat{b}_{ij}^t y_{ij}^t p_{ij}^t\right), \quad \forall t \in T, \forall i \in I \tag{5}$$

$$\left(a_{ij}^t - b_{ij}^t p_{ij}^t\right) - \left(\hat{a}_{ij}^t - \hat{b}_{ij}^t p_{ij}^t\right) \geq 0, \quad \forall t \in T, \forall i \in I, \forall j \in I \tag{6}$$

$$\underline{p}_{ij} \leq p_{ij}^t \leq \bar{p}_{ij}, \quad \forall t \in T, \forall i \in I, \forall j \in I \tag{7}$$

$$n_i^t \geq 0, \quad \forall t \in T, \forall i \in I \tag{8}$$

$$K \in \mathbb{N}^{+} \tag{9}$$

Objective Function (2) maximizes the operating profit. $\sum_{t\in T}\sum_{i\in I}\sum_{j\in I} p_{ij}^t\left(a_{ij}^t - b_{ij}^t p_{ij}^t\right)$ represents the revenue from serving users, and c_0K represents the depreciation cost of the bikes deployed in the systems. Constraint (3) determines the total number of bikes required to be deployed.

Constraint (4) ensures the flow conservation.$\sum_{j\in I}\left(a_{ij}^t - b_{ij}^t p_{ij}^t\right) + \sum_{j\in I}\left(\hat{a}_{ij}^t x_{ij}^t - \hat{b}_{ij}^t y_{ij}^t p_{ij}^t\right)$ represents the actual number of shared bikes riding from area i at period t, and $\sum_{j\in I}\left(a_{ij}^t - b_{ij}^t p_{ij}^t\right) + \sum_{j\in I}\left(\hat{a}_{ij}^t x_{ij}^t - \hat{b}_{ij}^t y_{ij}^t p_{ij}^t\right)$ represents the actual number of shared bikes riding to area i at period t. Constraint (5) guarantees that the total (actual) number of shared bikes riding from area i at period t is not larger than the number of shared bikes available at area $i \in I$ at the beginning of period t.

Constraint (6) ensures the non-negativity of the demand volume. Note that Constraint (6) is a relaxed constraint because the maximum volume of demand $\hat{a}_{ij}^t - \hat{b}_{ij}^t p_{ij}^t$ caused by the uncertainty is considered. Constraints (7)-(9) restrict the ranges of the variables in the upper level, where $\underline{p}_{ij}$ and $\overline{p}_{ij}$ are the given lower and upper limits of the price for traveling from area i to area j.

C. Lower level

The lower level formulates the decision of the virtual adversary that selects the worst scenario for the objective of the platform. The lower level can be expressed as the following mixed-integer nonlinear programming model:

$$\min \sum_{t\in T}\sum_{i\in I}\sum_{j\in I} p_{ij}^t\left(\hat{a}_{ij}^t x_{ij}^t - \hat{b}_{ij}^t y_{ij}^t p_{ij}^t\right) \tag{10}$$

$$-1 \le x_{ij}^t \le 1, \quad \forall t \in T, \forall i \in I, \forall j \in I \tag{11}$$

$$-1 \le y_{ij}^t \le 1, \quad \forall t \in T, \forall i \in I, \forall j \in I \tag{12}$$

Objective Function (10) minimizes the revenue variation caused by the uncertainty, because the actual revenue from serving users is $\sum_{t\in T}\sum_{i\in I}\sum_{j\in I} p_{ij}^t\left(a_{ij}^t - b_{ij}^t p_{ij}^t\right) + \sum_{t\in T}\sum_{i\in I}\sum_{j\in I} p_{ij}^t\left(\hat{a}_{ij}^t x_{ij}^t - \hat{b}_{ij}^t y_{ij}^t p_{ij}^t\right)$.Constraints (11)-(12) restrict the ranges of variables in the lower level.

IV. AN IMPROVED ROBUST METHOD

Model BL-RP pursues the optimal solutions for the worst-case scenario, and suffers a significant loss in the objective value. This section develops an improved robust method to trade off the robustness of the solution against the loss in the objective value. First, Section IV.A reformulates Model BL-RP into a single-level one. Section IV.B proposes constraint relaxations to obtain solutions robust against coefficient variations while avoiding significant losses in the objective value.

A. Single-level reformulation

The lower level of Model BI-RP is feasible and bounded, and thus its duality model attains the same optimal objective value, according to the strong duality theorem. Thus, we replace the lower level equivalently by the following dual model for tractability:

$$\max \sum_{t\in T}\sum_{i\in I}\sum_{j\in I} -\left(\lambda_{ij}^{t+} + \lambda_{ij}^{t-} + \mu_{ij}^{t+} + \mu_{ij}^{t-}\right) \tag{13}$$

$$\lambda_{ij}^{t+} - \lambda_{ij}^{t-} = \hat{a}_{ij}^t p_{ij}^t, \quad \forall t \in T, \forall i \in I, \forall j \in I \tag{14}$$

$$\mu_{ij}^{t+} - \mu_{ij}^{t-} = -\hat{b}_{ij}^t \left(p_{ij}^t\right)^2, \quad \forall t \in T, \forall i \in I, \forall j \in I \tag{15}$$

$$\lambda_{ij}^{t+} \ge 0,\ \lambda_{ij}^{t-} \ge 0,\ \mu_{ij}^{t+} \ge 0,\ \mu_{ij}^{t-} \ge 0, \quad \forall t \in T, \forall i \in I, \forall j \in I \tag{16}$$

λ_{ij}^{t+} and λ_{ij}^{t-} are the dual variables of the constraints $x_{ij}^t \ge -1$ and $-x_{ij}^t \ge -1$, respectively. μ_{ij}^{t+} and μ_{ij}^{t-} are the dual variables of the constraints $y_{ij}^t \ge -1$ and $-y_{ij}^t \ge -1$, respectively.

Constraints (13)-(16) are equivalent to the following objective function and constraints:

$$\max \sum_{t\in T}\sum_{i\in I}\sum_{j\in I} -p_{ij}^t\left(\hat{a}_{ij}^t - \hat{b}_{ij}^t p_{ij}^t\right) - 2\left(\lambda_{ij}^{t-} + \mu_{ij}^{t-}\right) \tag{17}$$

$$\lambda_{ij}^{t-} \ge 0,\ \mu_{ij}^{t-} \ge 0, \quad \forall t \in T, \forall i \in I, \forall j \in I \tag{18}$$

Both λ_{ij}^{t-} and μ_{ij}^{t-} are non-negative variables. Therefore, Objective Function (17) achieves the maximum value with $\lambda_{ij}^{t-} = 0$ and $\mu_{ij}^{t-} = 0$.

Therefore, Model BI-RP is reformulated equivalently into a mixed-integer quadratic programming model (called Model MIQP), with the following objective function:

$$\max\ F_1 = \sum_{t\in T}\sum_{i\in I}\sum_{j\in I} p_{ij}^t\left(a_{ij}^t - b_{ij}^t p_{ij}^t\right) - \sum_{t\in T}\sum_{i\in I}\sum_{j\in I} p_{ij}^t\left(\hat{a}_{ij}^t - \hat{b}_{ij}^t p_{ij}^t\right) - c_0K \tag{19}$$

and Constraints (3)–(9).

B. Constraint relaxation

Model MIQP can be solved directly by commercial solvers. However, it suffers a significant loss in the objective value, because the solution in the worst-case scenario is pursued, as in the traditional robust optimization methods. We introduce the restrictive coefficients M_i^t and N_i^t for each combination of $i \in I$ and $t \in T$ to balance the robustness of the solution and the loss in the objective value. Specifically, M_i^t refers to the maximum deviation between the actual number of bikes and the estimated number of bikes riding from area i at period t. N_i^t refers to the maximum deviation between the actual number of bikes and the estimated number of bikes riding to area i at period t. We have:

$$\left|\sum_{j\in I}\left(\hat{a}_{ji}^t - \hat{b}_{ji}^t y_{ji}^t p_{ji}^t\right)\right| \le M_i^t, \quad \forall i \in I, \forall t \in T \tag{20}$$

$$\left|\sum_{j\in I}\left(\hat{a}_{ji}^t x_{ji}^t - \hat{b}_{ji}^t y_{ji}^t p_{ji}^t\right)\right| \le N_i^t, \quad \forall i \in I, \forall t \in T \tag{21}$$

Constraint (20) restricts that the absolute deviation between the actual number of bikes riding from area i at period t and the estimated number of bikes riding from area i at period t is not larger than the maximum deviation M_i^t Constraint (21) restricts that the absolute deviation between the actual number of bikes riding to area i at period t and the estimated number of bikes riding to area i at period t is not larger than the maximum deviation N_i^t .The allowed ranges of M_i^t and N_i^t are $\left[0, \sum_{j\in I}\left|\left(\hat{a}_{ij}^t + \hat{b}_{ij}^t\bar{p}_{ij}\right)\right|\right]$ and $\left[0, \sum_{j\in I}\left|\left(\hat{a}_{ji}^t + \hat{b}_{ji}^t\bar{p}_{ij}\right)\right|\right]$, respectively. The magnitudes of M_i^t and N_i^t reflect the degree of awareness for the uncertain coefficients in Eq. (1). A value of zero indicates that the platform possesses complete information on the uncertain coefficients. Conversely, larger values of M_i^t and N_i^t correspond to a lower degree of information awareness.

Furthermore, we relax Constraints (4)-(5) according to Constraints (20)-(21) with the following constraints:

$$n_i^{t+1} \geq n_i^t - \sum_{j\in I}\left(a_{ij}^t - b_{ij}^t p_{ij}^t\right) - M_i^t + \sum_{j\in I}\left(a_{ji}^t - b_{ji}^t p_{ji}^t\right) - N_i^t, \quad \forall t \in T \setminus \{|T|\},\ \forall i \in I \tag{22}$$

$$n_i^{t+1} \leq n_i^t - \sum_{j\in I}\left(a_{ij}^t - b_{ij}^t p_{ij}^t\right) + M_i^t + \sum_{j\in I}\left(a_{ji}^t - b_{ji}^t p_{ji}^t\right) + N_i^t, \quad \forall t \in T \setminus \{|T|\},\ \forall i \in I \tag{23}$$

$$n_i^t \geq \sum_{j\in I}\left(a_{ij}^t - b_{ij}^t p_{ij}^t\right) - M_i^t, \quad \forall t \in T,\ \forall i \in I \tag{24}$$

Constraint (22) is obtained by relaxing Constraint (4), with $-\sum_{j\in I}\left(\hat{a}_{ij}^t x_{ij}^t - \hat{b}_{ij}^t y_{ij}^t p_{ij}^t\right)$ and $\sum_{j\in I}\left(\hat{a}_{ji}^t x_{ji}^t - \hat{b}_{ji}^t y_{ji}^t p_{ji}^t\right)$ replaced by $-M_i^t$ and $-N_i^t$, respectively. Similarly, Constraint (23) is obtained by relaxing Constraint (4), with $-\sum_{j\in I}\left(\hat{a}_{ij}^t x_{ij}^t - \hat{b}_{ij}^t y_{ij}^t p_{ij}^t\right)$ and $\sum_{j\in I}\left(\hat{a}_{ji}^t x_{ji}^t - \hat{b}_{ji}^t y_{ji}^t p_{ji}^t\right)$ replaced by M_i^t and N_i^t. Constraint (24) is obtained by relaxing Constraint (5), with $\sum_{j\in I}\left(\hat{a}_{ij}^t x_{ij}^t - \hat{b}_{ij}^t y_{ij}^t p_{ij}^t\right)$ replaced by $-M_i^t$.

In this way, Model MIQP is transformed into a relaxed model (called Model MIQP-RE), with Objective Function (19), Constraints (3), (6)-(9) and (22)-(24). Compared to Model MIQP, Model MIQP-RE can achieve an optimal solution within a shrunk variation range of the coefficients in the demand function, by allowing only partial constraints regarding the whole variation range of the coefficients to be satisfied.

V. VALIDATION AND EVALUATION

This section validates the proposed method based on extensive numerical experiments. The settings of instances are presented in Section V.A, followed by the test of the computational efficiency of Model MIQP-RE in Section V.B. The performances of Model MIQP-RE and the deterministic model without considering the uncertainty in the demand are compared in Section V.C. The effects of the type of uncertainty and the price elasticity of demand are investigated in Section V.D and Section V.E, respectively.

All the experiments were executed on a personal computer equipped with an 8-core CPU (3.80 GHz) and 16.0 GB RAM. Model MIQP-RE was solved using IBM ILOG CPLEX (version 12.6.1) embedded in Visual Studio 2013 with a working memory of 2048 MB, where the “depth-first search” strategy was applied.

A. Settings of the instances

The instances were generated according to [9] and [23]. Three scales (i.e., small, medium, and large) of synthetic instances were considered, and each scale contains four instances. The instances were named by scale and number (such as S1, M1 and L1). The service region was squared with the side lengths of 7 km, 11 km, and 13 km in the three scales of instances, respectively, and divided into 50,100, and 150 areas using Voronoi partition. The riding distance between two areas is the distance between their centers of mass multiplied by a random real number in the range of [1.5,2.5]. The planning horizon was divided into four periods, i.e., morning peak hours, daytime off-peak hours, evening peak hours, and evening off-peak hours.

The means of the estimated total maximum base of users for all O-D pairs in the three scales of instances are shown in Table II. In each period, the estimated maximum base of users with the trip lengths smaller than 4.5 km, smaller than 9 km but larger than 4.5 km, and larger than 9 km accounted for 75%, 15%, and 5%, respectively; the estimated maximum base of users with the destination consistent with the origin accounted for 5%, based on the findings of [23]. The estimated maximum base of users for each origin-destination pair in each period followed the Poisson distribution. The fluctuation volume for the maximum base of users for each O-D pair in each period was a random integer in the set of $\{1, 2\}$.

For all O-D pairs in the planning horizon, the estimated price elasticity of demand b_{ij}^t and the fluctuation volume for the price elasticity of demand $\hat{b}_{ij}^t$ were set as 3.0 vehicle/¥ and 0.2 vehicle/¥ respectively, according to a questionnaire investigation conducted in Shenyang, China, in 2025. The lower limits of the price were set as ¥ 0 for all O-D pairs. The upper limits of the price for the trips smaller than 4.5 km, smaller than 9 km but larger than 4.5 km, and larger than 9 km were ¥2, ¥4, and ¥6, respectively. The upper limit of the price for the trips with the destination consistent with the origin was ¥6. The depreciation cost c_0 was ¥0.3. The parameters M_i^t and N_i^t were set to $0.05\sum_{j\in I}\left(\hat{a}_{ij}^t + \hat{b}_{ij}^t\bar{p}_{ij}\right)$ and $0.05\sum_{j\in I}\left(\hat{a}_{ji}^t + \hat{b}_{ji}^t\bar{p}_{ji}\right)$, respectively.

B. Computational efficiency of Model MIQP-RE

The operations corresponding to the solutions obtained by Model MIQP-RE and the deterministic model were imitated using Monte Carlo simulation. The deterministic model, replaces all the uncertain parameters in Model TLMIP with their nominal values. Specifically, the deterministic model adopts Objective Function (2) as its objective function and includes Constraints (3), (6)–(9), together with the following constraints.

TABLE II Means Of Estimated Total Maximum Base Of Users

Scale	Morning peak hours	Daytime off-peak hours	Evening peak hours	Evening off-peak hours
Small	20,000	8,000	16,000	8,000
Medium	80,000	32,000	64,000	32,000
Large	100,000	40,000	80,000	40,000

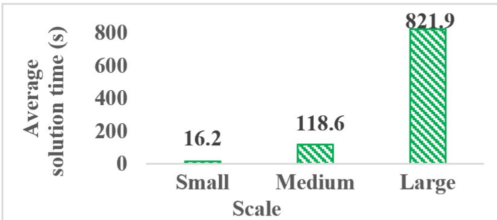


Fig. 2. Average solution times for the instances

$$n_i^{t+1} = n_i^t - \sum_{j \in I} \left(a_{ij}^t - b_{ij}^t p_{ij}^t\right)$$

$$+ \sum_{j \in I} \left(a_{ji}^t - b_{ji}^t p_{ji}^t\right), \quad \forall t \in T \setminus \{|T|\}, \forall i \in I \tag{25}$$

$$n_i^t \geq \sum_{j \in I} \left(a_{ij}^t - b_{ij}^t p_{ij}^t\right), \quad \forall t \in T, \forall i \in I \tag{26}$$

$$\left(a_{ij}^t - b_{ij}^t p_{ij}^t\right) \geq 0, \quad \forall t \in T, \forall i \in I, \forall j \in I \tag{27}$$

Constraints (25)–(27) correspond to Constraints (4)–(6), respectively. The deterministic model can be easily solved by CPLEX.

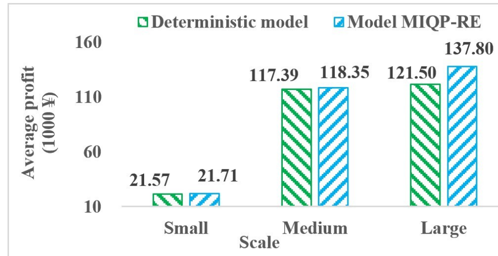
In the simulation execution, the uncertainty in demand was considered, where x_{ij}^t and y_{ij}^t were treated as uncertain coefficients. We assumed that both x_{ij}^t and y_{ij}^t were uniformly distributed in the range of $[-1, 1]$. The simulation was run 2000 times independently for each solution of each model, with the mean of the performance measure adopted as the final result. The following performance measures are considered.

a) The profit is defined as the actual revenue obtained by the simulation over the planning horizon minus the depreciation cost of the deployed bikes.

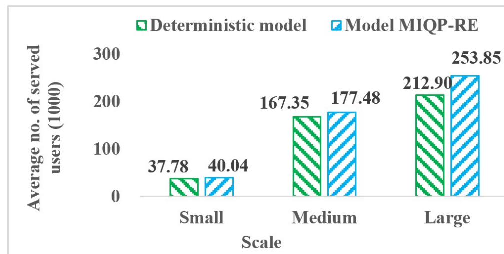
b) The number of served users is the actual total number of satisfied orders obtained by the simulation over the planning horizon.

In the simulation execution, if the supply of bikes at an area in a period cannot cover the volume of demand at the area, the demand riding to each area is satisfied proportionally, as formulated in [24]. The comparative results between the deterministic model and Model MIQP-RE are shown in Table III.

For each instance, there is always a simultaneous increment in the profit and the number of served users obtained by Model MIQP-RE compared to those resulting from the deterministic model. The profit increases by 0.61%, 0.69%, and 13.11% for the small-, medium-, and large-scale instances, respectively; the number of served users increases by 5.86%, 5.88%, and 18.63%, respectively. The consistent increments in the three scales indicate that Model MIQP-RE can cope well with uncertainties in the operations. The larger increments in the large-scale instances suggest that the benefits of the proposed method are particularly pronounced in more complex systems, where the deterministic model may fail to account for compounding uncertainties.



(a) Average profit



(b) Average number of served users

Fig. 3. Effect of the type of uncertainty

C. Effect of the type of uncertainty

The effect of the type of uncertainty is investigated in this section. Besides the baseline scenario in Section V.C, where both x_{ij}^t and y_{ij}^t were assumed to be uniformly distributed in the range of $[-1, 1]$, the following scenario was considered.**Scenario I:** x_{ij}^t and y_{ij}^t were both normally distributed with the mean of zero and the standard deviation of 0.3, and bounded in the range of $[-1, 1]$.

For each instance, the Monte Carlo simulation was run in Scenario I under the solutions of Model MIQP-RE and the deterministic model. The average results of the performance measures under the deterministic model and Model MIQP-RE are compared for each scale in Fig. 3.

As shown in Fig. 3, for each scale in Scenario I, both the average profit and the average number of served users achieved by Model MIQP-RE are improved compared to those obtained by the deterministic model. The superiority of Model MIQP-RE is more obvious for the large-scale instances, as discovered in Section V.D. The consistency of the results under the baseline scenario and Scenario I demonstrates the robustness of the proposed method.

TABLE III System Performances Under The Deterministic Model And Model MIQP-RE

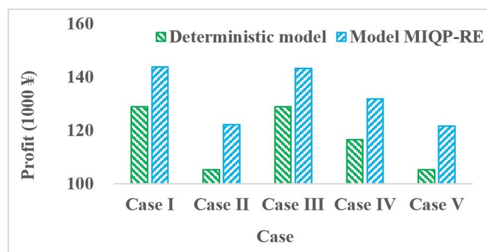
Scale	Instance	Deterministic model		Model MIQP-RE			
		Profit (¥)	No. of served users	Profit (¥)	Imp. PR[a] (%)	No. of served users	Imp.NoSU[b] (%)
Small	S1	21,946	37,557	22,059	0.52	39,752	5.85
	S2	18,932	37,679	19,022	0.48	39,888	5.86
	S3	23,123	38,145	23,333	0.90	40,410	5.94
	S4	22,315	37,919	22,437	0.55	40,111	5.78
	Average				0.61		5.86
Medium	M1	116,299	167,013	117,163	0.74	176,822	5.87
	M2	117,220	167,058	118,194	0.83	176,925	5.91
	M3	119,106	168,521	120,255	0.96	178,741	6.06
	M4	117,016	167,740	117,784	0.66	177,422	5.77
	Average				0.69		5.88
Large	L1	117,111	211,924	133,897	14.33	252,278	19.04
	L2	116,812	212,686	132,208	13.18	251,275	18.14
	L3	124,959	214,958	141,623	13.34	255,348	18.79
	L4	128,550	216,324	143,452	11.59	256,426	18.54
	Average				13.11		18.63

[a] "Imp.PR": improvement in the profit compared to the profit obtained by the deterministic model.
[b] "Imp.NoSU": improvement in the number of served users compared to that obtained by the deterministic model.

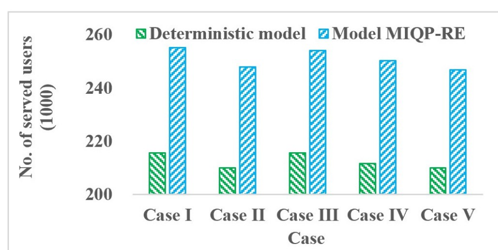
D. *Effect of the price elasticity of demand*

Instance L2 was selected for sensitivity analysis on the price elasticity of demand and its fluctuation volume. We solved the deterministic model and Model MIQP-RE separately under the following additional five cases, with the other parameters remaining unchanged: Case I: $b_{ij}^t = 2.5$ vehicle/¥, $\hat{b}_{ij}^t = 0.2$ vehicle/¥;
Case II: $b_{ij}^t = 3.5$ vehicle/¥, $\hat{b}_{ij}^t = 0.2$ vehicle/¥;
Case III: $b_{ij}^t = 2.5$ vehicle/¥, $\hat{b}_{ij}^t = 0.3$ vehicle/¥;
Case IV: $b_{ij}^t = 3.0$ vehicle/¥, $\hat{b}_{ij}^t = 0.3$ vehicle/¥;
Case V: $b_{ij}^t = 3.5$ vehicle/¥, $\hat{b}_{ij}^t = 0.3$ vehicle/¥.
The solutions obtained by the two models were separately evaluated using the simulation in the baseline scenario. The results of the profit and the number of served users are presented in Fig. 4.

As can be seen, both the profit and the number of served users decrease with the increment in the price elasticity of demand or its fluctuation volume. The results indicate that large elasticities of demand are harmful for the application of dynamic prices. However, remarkable improvements in the profit and the number of served users can always be guaranteed within the investigated variation range of the price elasticity of demand and its fluctuation volume, which also demonstrates the robustness of Model MIQP-RE.



(a) Profit



(b) Number of served users

Fig. 4. Effect of the elasticity of demand and its fluctuation volume

VI. CONCLUSIONS AND FURTHER RESEARCH

This research investigates an integrated fleet sizing and dynamic pricing problem for bike-sharing systems. A bi-level mixed-integer programming model is established under the robust optimization framework. The model is reformulated into a mixed-integer quadratic programming model based on the strong duality theorem. Constraint relaxations are introduced to trade off the robustness of the solution and the loss of the objective value.

The proposed methods are validated and evaluated using numerical experiments. Results demonstrate that the mixed-integer quadratic programming model is applicable in practice in terms of computational efficiency, and that the decisions obtained by the model can increase the profit and the number of served users simultaneously. What's more, the superiority of the proposed methods is independent of the type of uncertainty and the price elasticity of demand.

This research could be extended in several aspects. First, a more complicated relationship (rather than a simple linear function) between the trip price and the volume of demand could be considered for the price optimization. Second, tailored algorithms can be developed to obtain solutions for especially large-scale instances quickly, such that the scalability of the proposed methods is expanded.

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Haiyu Kan received the Ph.D. degree in systems engineering from Northeastern University, Shenyang, China, in 2024. She is currently a lecturer with the School of Management, Shenyang University of Technology, China. Her research interests include modeling and optimization, logistics and transportation.

Chengqing Yu received the B.S. degree in engineering management from Wuhan College, Wuhan, China, in 2024. He is currently pursuing a master in the School of Management, Shenyang University of Technology, China. His main research interests include and vehicle routing problems and intelligent optimization algorithms.

Ruiyou Zhang (M'20) received the B.Sc. Degree in Automation and the Ph.D. Degree in Systems Engineering from Northeastern University, Shenyang, China, in 2002 and 2007, respectively. From 2007 to 2008, he was a post-doctoral fellow with Department of Industrial Engineering, Pusan National University, Busan, South Korea. From 2010 to 2011, he was a visiting scholar with School of Industrial and Systems Engineering, Georgia Institute of Technology, Georgia, USA.

He is currently a Long-termed Professor with College of Information Science and Engineering, Northeastern University, Shenyang, China. He has published more than 50 refereed technical papers on international / domestic journals. His major research interests include modeling and optimization, intelligent manufacturing, logistics and transportation, service science and engineering.

Peng Liu received the Ph.D. degree in systems engineering from Northeastern University, in January 2009. He is currently a Professor and a Doctoral Supervisor with the School of Management, Shenyang University of Technology, China. He is in charge of the National Natural Science Foundation of China and the Program for Liaoning Excellent Talents in University. He has published a monograph and more than 40 papers in international journals such as Computers & Industrial Engineering, Swarm and Evolutionary Computation, International Journal of Computer Integrated Manufacturing, International Journal of Management Science and Engineering Management, Applied Mathematical Modelling, Applied Mathematics and Computation, International Journal of Advanced Manufacturing Technology. His papers were cited over 400 times according to WoS. His research interests include production scheduling, logistics optimization and combinatorial optimization.