

Improved Walrus Optimizer for Feature Mode Decomposition Parameter Selection in Bearing Fault Feature Extraction

Qimei Zhan, Xiangkun Song, and Jian Zhao

Abstract—Feature Mode Decomposition (FMD) is sensitive to parameter settings, which are often selected empirically and may limit bearing fault feature extraction. This study proposes an Improved Walrus Optimizer-assisted FMD (IWO-FMD) framework for bearing fault feature enhancement. The framework is designed for approximately fixed-speed conditions with known fault characteristic frequencies. IWO-FMD jointly optimizes FilterSize, CutNum, ModeNum, and MaxIterNum. Parameter search and held-out evaluation are performed on separate, non-overlapping vibration segments. Experiments on 12 kHz drive-end signals from the Case Western Reserve University (CWRU) bearing dataset cover 20 load-fault conditions with ten repeated runs per condition. IWO-FMD is compared with fixed-parameter FMD and five optimizer-assisted FMD methods. All optimizer-assisted methods use identical parameter ranges and 100 true objective-function evaluations. Compared with fixed-parameter FMD, IWO-FMD achieves higher mean fault characteristic peak (FCP) and signal-to-noise ratio (SNR) values in all 20 conditions and higher mean correlated kurtosis (CK) values in 19 conditions. After Benjamini-Hochberg correction, the SNR and CK differences are statistically significant in 19 and 17 conditions, respectively. The pooled fault-detection rate increases from 45.0% to 60.5%, while the candidate-level false-alarm rate decreases from 10.8% to 7.5%. Overall, IWO-FMD improves FMD parameter selection and provides a better balance between bearing fault feature enhancement and false-alarm control.

Index Terms—Bearing fault feature extraction, Fault characteristic frequency, Feature mode decomposition, FMD parameter optimization, Improved Walrus Optimizer.

I. INTRODUCTION

ROTATING machinery is widely used in modern industrial production systems. As critical components of such systems, bearings can develop faults under long-term alternating loads and friction. In practice, bearing faults commonly affect the inner race, outer race, and rolling elements [1]. Bearing damage is a major source of failure in rotating machinery and

can reduce equipment reliability and increase safety risks [2]. Therefore, accurate and timely bearing fault feature extraction is essential for predictive maintenance and industrial safety.

Bearing vibration signals are often non-stationary and contain background noise and coupled vibration components. These factors may obscure impulsive fault features, particularly when the fault-related response is weak, making reliable feature extraction challenging [3]. Multi-modal fusion of vibration and profile measurements has improved robustness in closely related monitoring tasks [4]. Signal decomposition methods are widely used to address this problem. Among them, Feature Mode Decomposition (FMD) [5] has attracted attention because of its decomposition efficiency and adaptability to non-stationary signals. Other commonly used signal decomposition methods include Empirical Mode Decomposition (EMD) [6], Ensemble Empirical Mode Decomposition (EEMD) [7], Local Mean Decomposition (LMD) [8], Empirical Wavelet Transform (EWT) [9], and Variational Mode Decomposition (VMD) [10]. Among these methods, FMD uses adaptive filtering and mode selection to separate signal components and preserve fault-related information. However, the effectiveness of this decomposition process remains closely related to the selected FMD parameters.

In particular, FMD performance depends strongly on filter length, frequency-band segment number, mode number, and iteration number. These parameters are commonly selected through empirical settings, whose reliability may decrease when operating loads and fault characteristics change. Inappropriate parameter combinations may cause mode mixing, weak fault-related components, or insufficient suppression of irrelevant signal content [11]. Sensitivity to parameter settings and external disturbances has also been reported [12]. Consequently, adaptive FMD parameter selection is required to reduce the dependence of FMD on fixed empirical settings.

One practical approach to reducing the empirical dependence of FMD is to introduce metaheuristic parameter search, which has likewise proven effective for high-dimensional feature selection [13]. Recent research has demonstrated the feasibility of this approach by combining an improved Hippo Optimization algorithm with FMD for bearing fault feature extraction [14]. In that method, the FMD mode number and filter length are optimized using envelope entropy, and the target mode is selected according to kurtosis. However, only two FMD parameters are optimized. In addition, the separation between parameter search and held-out evaluation is not explicitly addressed. These limitations motivate extending

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optimizer-assisted FMD to a larger discrete and constrained parameter space under a limited objective-function evaluation budget.

Metaheuristic algorithms (MAs) provide gradient-free search mechanisms for complex parameter spaces. However, the No-Free-Lunch theorem indicates that no optimizer can consistently dominate all problem classes [15]. The suitability of an optimizer therefore depends on the target problem. This has motivated extensive work adapting population-based optimizers to discrete combinatorial problems, including migratory-bird search for disassembly line balancing [16] and improved grey wolf search for collaborative disassembly [17]. Reinforcement-learning, hyper-heuristic, and multi-objective formulations have also been applied to comparable disassembly and scheduling tasks [18]–[21]. Among these methods, the Walrus Optimizer (WO), proposed by Han et al. [22], adopts a relatively simple population-based search mechanism. However, its population diversity may decrease during iteration, increasing the risk of premature convergence. Recent WO variants have therefore focused on improving population diversity, search guidance, and local refinement [23]–[25]. Accordingly, this study develops an Improved Walrus Optimizer (IWO) for the discrete and constrained FMD parameter space.

Based on IWO, an IWO-assisted FMD (IWO-FMD) framework is proposed for bearing fault feature enhancement. The framework is designed for approximately fixed-speed conditions with known fault characteristic frequencies. IWO searches for FilterSize, CutNum, ModeNum, and MaxIterNum, whereas FMD performs signal decomposition. These four parameters affect mode separation, the response around the fault characteristic frequency, impulsive-feature preservation, and computational cost. To improve experimental reliability, parameter search and held-out evaluation are conducted on non-overlapping vibration segments. The fault characteristic peak (FCP) is used as the optimization response, whereas the signal-to-noise ratio (SNR) and correlated kurtosis (CK) serve as auxiliary evaluation indicators.

This paper makes three main contributions:

- (1) An IWO-FMD framework is developed for adaptive FMD parameter selection. The framework reduces dependence on fixed empirical settings. It is designed for bearing fault feature enhancement under approximately fixed-speed conditions with known fault characteristic frequencies.
- (2) Four discrete and constrained FMD parameters, namely FilterSize, CutNum, ModeNum, and MaxIterNum, are jointly optimized. Deterministic mapping and constraint-repair rules ensure that each candidate parameter combination remains within the feasible search space.
- (3) A held-out evaluation protocol is adopted, in which parameter search and held-out evaluation use non-overlapping signal segments. FCP serves as the optimization response, whereas SNR and CK are used as auxiliary evaluation indicators. Multiple optimizer-assisted FMD methods are compared under the same objective-function evaluation budget. A calibrated false-alarm and fault-detection analysis is also conducted using normal and fault records.

The remainder of this paper is organized as follows. Section II introduces the WO framework and the five improvement strategies of IWO. In Section III, IWO is evaluated using the CEC2017 benchmark suite and constrained engineering design problems, followed by an ablation study. Section IV presents the IWO-FMD framework, including parameter representation, constraint handling, and the FCP-based fitness function. Bearing-signal experiments and the corresponding results are reported and discussed in Section V. Conclusions and future research directions are presented in Section VI.

II. IMPROVED WALRUS OPTIMIZER

This section reviews the WO search framework and presents the five improvement strategies introduced in IWO. IWO retains the danger and safety signals used for behavior selection. It modifies the population structure and selected position-update rules. These changes are intended to enhance population diversity, search guidance, and late-stage refinement.

A. Overview of WO

WO is inspired by the collective behavior of walrus herds and uses a population of walruses to explore the search space. Each walrus represents a candidate solution encoded by a vector of decision-variable values. The positions are updated iteratively, while the best solution found during the search is retained.

At the beginning of the search, WO initializes N candidate solutions within the prescribed bounds. The resulting population matrix is defined as follows:

$$X = LB + rand_1 \cdot (UB - LB) \quad (1)$$

$$X = \begin{bmatrix} x_{1,1} & x_{1,2} & \cdots & x_{1,D} \\ x_{2,1} & x_{2,2} & \cdots & x_{2,D} \\ \vdots & \vdots & \ddots & \vdots \\ x_{N,1} & x_{N,2} & \cdots & x_{N,D} \end{bmatrix}_{N \times D} \quad (2)$$

where X denotes the population matrix, LB and UB are the lower and upper bound vectors of the search space, respectively. $rand_1$ represents uniformly distributed random numbers in $[0, 1]$ used for population initialization. N is the population size, and D is the problem dimension.

After initialization, the population is divided into adult and juvenile walruses. Adult walruses account for 90% of the population, whereas juvenile walruses account for 10%. The adult group contains equal numbers of male and female walruses. WO uses the danger and safety signals to determine the active search behavior at each iteration.

$$Danger_signal = A \cdot R \quad (3)$$

$$\alpha = 1 - \frac{t}{MaxIter} \quad (4)$$

$$A = 2 \cdot \alpha \quad (5)$$

$$R = 2 \cdot rand_4 - 1 \quad (6)$$

$$Safety_signal = rand_5 \quad (7)$$

where *Danger_signal* and *Safety_signal* denote the danger signal and safety signal, respectively. *A* represents the danger intensity factor, and *R* represents the risk disturbance factor. α decreases gradually from 1 to 0 with the number of iterations *t*, and *MaxIter* denotes the maximum number of iterations. *rand₄* and *rand₅* are independent random numbers uniformly distributed in (0, 1), respectively.

The search process consists of exploration and exploitation. Migration constitutes the exploration phase. Exploitation includes roosting and foraging. Foraging is further divided into fleeing and gathering. Migration is performed when $|Danger_signal| \geq 1$. Roosting is performed when $|Danger_signal| < 1$ and $Safety_signal \geq 0.5$. Fleeing is performed when $0.5 \leq |Danger_signal| < 1$ and $Safety_signal < 0.5$. Gathering is performed when $|Danger_signal| < 0.5$ and $Safety_signal < 0.5$. The detailed position-update rules for these behaviors are provided (see Appendix A).

B. Development of IWO

IWO retains the behavior-selection framework of WO and introduces five improvement strategies. Population restructuring changes the population roles. Spiral-enhanced male migration modifies migration, while elite-guided juvenile search modifies fleeing. Adaptive dual-candidate fusion modifies gathering. Elite perturbation introduces additional variation during the later search stage. The danger and safety signals and the roosting rules remain unchanged.

1) *Population restructuring*: In WO, male and female walruses each account for 45% of the population, whereas juvenile walruses account for only 10%. Therefore, only a small proportion of individuals follow the juvenile-specific update rules. To increase juvenile participation, IWO assigns nominal proportions of 22.5%, 22.5%, and 55% to the male, female, and juvenile groups, respectively. The numbers of male and female walruses are rounded to integers, and the remaining individuals are assigned to the juvenile group. This adjustment preserves all three population roles while increasing the use of juvenile-specific search. Table I compares the population structures of WO and IWO.

2) *Spiral-enhanced male migration*: During migration, WO updates each walrus using a displacement based on the position difference between two randomly selected sentinel walruses. This difference-based update provides the basic exploration step. However, it may provide limited variation in the migration trajectory. IWO therefore adds a dynamic spiral displacement to the male walruses while retaining the WO migration step. Female and juvenile walruses continue to follow the WO migration rule (see Eqs. (A1) – (A3) in Appendix A). The spiral-enhanced male migration is defined as follows:

$$\rho = 0.5 - 0.3 \cdot \frac{t}{MaxIter} \quad (8)$$

$$\psi = -1 + 0.5 \cdot \frac{t}{MaxIter} \quad (9)$$

$$S_{i,j}^t = \rho \cdot e^{\psi \cdot \pi \cdot rand_2} \cdot \cos(\pi \cdot rand_3) \cdot (X_{rand,j}^t - Male_{i,j}^t) \quad (10)$$

$$Male_{i,j}^{t+1} = Male_{i,j}^t + Migration_step + S_{i,j}^t \quad (11)$$

where ρ and ψ are iteration-dependent coefficients that regulate the spiral displacement. *t* and *MaxIter* denote the current and maximum iteration numbers, respectively. $S_{i,j}^t$ is the spiral displacement. *rand₂* and *rand₃* are two independent random numbers uniformly distributed in (0, 1). $X_{rand,j}^t$ is the corresponding position component of a randomly selected walrus. $Male_{i,j}^t$ and $Male_{i,j}^{t+1}$ are the current and updated position components of the *i*-th male walrus in the *j*-th dimension. *Migration_step* denotes the WO migration displacement (see Eq. (A2) in Appendix A).

3) *Elite-guided juvenile search*: During fleeing, WO applies the same update rule to all walruses (see Eq. (A10) in Appendix A). This rule uses the current global best position but does not explicitly exploit the best male individuals. IWO retains the original fleeing rule for male and female walruses. Juvenile walruses are instead guided by the best male, the second-best male, and the current global best positions. The update mechanism is defined as follows:

$$d_1 = \|Male_{best,j}^t - X_{best,j}^t\| \quad (12)$$

$$d_2 = \|Male_{second,j}^t - X_{best,j}^t\| \quad (13)$$

where d_1 and d_2 denote the Euclidean distances from the current global best position to the best and second-best male walruses, respectively. $Male_{best,j}^t$ and $Male_{second,j}^t$ represent the position vectors of the best and second-best male walruses, respectively, and $X_{best,j}^t$ is the current global best position.

The normalized distance-based weight is defined as:

$$w = \frac{\frac{1}{d_1 + \varepsilon}}{\frac{1}{d_1 + \varepsilon} + \frac{1}{d_2 + \varepsilon}} \quad (14)$$

where *w* is the normalized weight, and ε is a small positive constant used to avoid division by zero. Thus, the male walrus closer to the current global best position receives a larger weight.

The male-driven leadership position and the juvenile update rule are expressed as follows:

$$L_j^t = w \cdot Male_{best,j}^t + (1 - w) \cdot Male_{second,j}^t \quad (15)$$

$$\eta = 0.6 - 0.2 \cdot \frac{t}{MaxIter} \quad (16)$$

TABLE I
POPULATION STRUCTURES OF WO AND IWO

Algorithm	Male walruses	Female walruses	Juvenile walruses
WO	45%	45%	10%
IWO	22.5%	22.5%	55%

$$Juvenile_{i,j}^{t+1} = \eta \cdot L_j^t + (1 - \eta) \cdot X_{best,j}^t \quad (17)$$

where L_j^t denotes the j -th component of the male-driven leadership position. η is an iteration-dependent coefficient that balances the male-driven leadership position and the current global best position. $Juvenile_{i,j}^{t+1}$ denotes the updated j -th position component of the i -th juvenile walrus.

4) *Adaptive dual-candidate fusion*: During gathering, WO generates two candidate positions from the current best and second-best positions (see Eqs. (A11) – (A15) in Appendix A). The two candidates are combined with equal weights. However, fixed equal weighting assigns the same contribution to both candidates throughout the search. IWO therefore introduces an iteration-dependent fusion weight. As the search proceeds, greater weight is gradually assigned to the candidate generated from the current best position. The update is defined as follows:

$$\varphi = 0.5 + 0.2 \cdot \frac{t}{MaxIter} \quad (18)$$

$$X_{i,j}^{t+1} = \varphi \cdot X_1 + (1 - \varphi) \cdot X_2 \quad (19)$$

where φ is the adaptive fusion weight. $X_{i,j}^{t+1}$ denotes the updated j -th position component of the i -th walrus. X_1 and X_2 denote the two temporary candidate values for the j -th position component (see Eqs. (A12) and (A13) in Appendix A).

5) *Elite perturbation*: WO does not apply an additional perturbation after the standard position updates. To introduce additional variation in the later search stage, IWO activates elite perturbation after 60% of the objective-function evaluation budget has been consumed. Successive perturbations are separated by at least five population-evaluation rounds. The number of elite individuals is defined as $N_e = \max(2, \text{round}(0.05N))$, where N is the population size. At each activation, the N_e individuals with the best current fitness values are selected. Since $N = 30$ in all experiments, $N_e = 2$. The elite center and perturbed candidates are then calculated as follows:

$$C_j^t = \frac{1}{N_e} \sum_{i=1}^{N_e} X_{i,j}^t, j = 1, 2, \dots, D \quad (20)$$

$$\begin{aligned} \tilde{X}_{i,j}^{t+1} &= C_j^t + \delta_{i,j}^t \\ , i &= 1, 2, \dots, N_e, j = 1, 2, \dots, D \end{aligned} \quad (21)$$

$$\begin{aligned} \delta_{i,j}^t &= 0.022 (UB_j - LB_j) \cdot z_{i,j}^t \\ , z_{i,j}^t &\sim \mathcal{N}(0, 1) \end{aligned} \quad (22)$$

where C_j^t is the j -th component of the elite center, and $\tilde{X}_{i,j}^{t+1}$ is the j -th component of the i -th perturbed candidate. $\delta_{i,j}^t$ denotes

the Gaussian perturbation, $z_{i,j}^t$ is an independently sampled standard Gaussian random variable, and LB_j and UB_j are the lower and upper bounds of the j -th decision variable, respectively. The coefficient 0.022 controls the perturbation magnitude relative to the search range.

The perturbed candidates replace the corresponding candidate positions before boundary control. They are evaluated as part of the regular population evaluation. Therefore, the perturbation step does not require additional objective-function evaluations.

6) *Overall procedure of IWO*: The five improvement strategies are incorporated into the WO framework to form IWO. Algorithm 1 presents the overall procedure and the execution sequence of its search behaviors. *FES* records the number of objective-function evaluations, including those of the initial population.

III. NUMERICAL VALIDATION OF IWO

The numerical performance of IWO is evaluated using unconstrained benchmark functions and constrained engineering design problems. The CEC2017 experiments evaluate optimization accuracy, stability, and robustness across different dimensions. The constrained engineering problems further evaluate IWO under practical constraint conditions. An ablation study is then conducted to examine the contribution of each improvement strategy.

A. General optimization performance

This section evaluates the numerical optimization performance of IWO using the CEC2017 test suite and four constrained engineering benchmark problems. IWO is compared with eight representative metaheuristic algorithms, namely WO, Grey Wolf Optimizer (GWO) [26], Whale Optimization Algorithm (WOA) [27], Particle Swarm Optimization (PSO) [28], Differential Evolution (DE) [29], Harris Hawks Optimization (HHO) [30], Snow Geese Algorithm (SGA) [31], and Hiking Optimization Algorithm (HOA) [32]. All algorithms are tested using the same population size, maximum number of iterations, and objective-function evaluation budget. The comparison focuses on solution accuracy, convergence behavior, stability, and statistical ranking under the tested settings.

1) *CEC2017 benchmark experiments*: The CEC2017 benchmark suite includes unimodal, multimodal, hybrid, and composition functions. Function F2 is excluded, leaving 29 test functions. All algorithms are tested at 30, 50, and 100 dimensions over 30 independent runs for each function and dimension. The population size is 30, the iteration limit is 500, and the maximum objective-function evaluation budget is 15,000 per run. The remaining parameters of the comparison algorithms follow their original studies. Table II summarizes

Algorithm 1: The pseudocode of IWO

Input: Population size N , dimension D , maximum iterations $MaxIter$, maximum objective-function evaluations Max_FES , and bounds LB and UB
Output: Global best solution X_{best} and its fitness $f(X_{best})$

```

1  Begin
2  Initialize population  $X$  using Eq. (2), and set  $FES = 0$ 
3  Evaluate the initial population and update  $FES$ ,  $X_{best}$  and  $X_{second}$ 
4  Divide  $X$  into male, female, and juvenile groups by Table I
5  Determine  $Male_{best,j}^t$  and  $Male_{second,j}^t$  from the male group
6  For  $t = 0$  to  $MaxIter - 1$  do
7      If  $FES \geq Max\_FES$  then break
8      Calculate  $Danger\_signal$  and  $Safety\_signal$  by Eqs. (3) – (7)
9      If  $|Danger\_signal| \geq 1$  then
10         Update female and juvenile walruses using the WO migration rule (see Eqs. (A1) – (A3) in Appendix A)
11         Update the male walruses using Eqs. (8) – (11)
12      Else
13         If  $Safety\_signal \geq 0.5$  then
14             Redistribute the male walruses using the Halton sequence
15             Update female and juvenile walruses using the WO roosting rules (see Eqs. (A4) – (A9) in Appendix A)
16         Else
17             If  $|Danger\_signal| \geq 0.5$  then
18                 Determine the best and second-best male walruses according to their fitness values
19                 Update male and female walruses using the WO fleeing rule (see Eq. (A10) in Appendix A)
20                 Update juvenile walruses using Eqs. (12) – (17)
21             Else
22                 Generate  $X_1$  and  $X_2$  using the WO gathering rules (see Eqs. (A12) – (A15) in Appendix A)
23                 Update all walruses using Eqs. (18) and (19)
24             End If
25         End If
26      End If
27      If the elite perturbation condition is satisfied then
28          Select the elite individuals from the evaluated population
29          Construct the elite center using Eq. (20)
30          Generate the perturbed candidates using Eqs. (21) and (22)
31          Replace the corresponding candidate positions with the perturbed candidates
32      End If
33      Apply boundary control to the candidate positions
34      Evaluate the candidate population within the remaining objective-function evaluation budget
35      Update  $FES$ ,  $X_{best}$ ,  $X_{second}$ 
36  End For
37  Return  $X_{best}$  and  $f(X_{best})$ 
38 End

```

the pairwise statistical outcomes (see Tables XIV–XVII and Figs. 6 and 7 in Appendix B). The symbols “+/-/=” indicate that IWO performs significantly better than, significantly worse than, or shows no significant difference from the corresponding comparison algorithm, respectively.

As shown in Table II, IWO records more wins than losses against all eight comparison algorithms across the three dimensions. IWO records particularly favorable outcomes against WOA, SGA, HOA, and HHO, whereas the comparisons with PSO and GWO are more mixed. Overall, IWO remains competitive across dimensions but does not dominate every test function.

2) *Constrained engineering benchmark problems:* Four constrained engineering benchmark problems are considered, including the three-bar truss, tension/compression spring, speed reducer, and car side impact design problems [33]–[36] (see Appendix C). All algorithms use the same population size, stopping criterion, boundary-handling strategy, and constraint-handling method. Table III summarizes the mean-based rankings of IWO on the four constrained engineering benchmark problems.

As shown in Table III, IWO ranks first on the three-bar truss, speed reducer, and car side-impact problems. For the tension/compression spring problem, IWO ranks second,

whereas GWO obtains the lowest mean objective value. These results indicate competitive constrained-search performance under the tested settings.

B. Ablation study

The ablation study evaluates the contribution of each IWO improvement strategy on the 50-dimensional CEC2017 benchmark suite. IWO-noS1, IWO-noS2, IWO-noS3, IWO-noS4, and IWO-noS5 remove population restructuring, spiral-enhanced male migration, elite-guided juvenile search, adaptive dual-candidate fusion, and elite perturbation, respectively. WO, full IWO, and the five ablated versions use the same population size, iteration limit, and objective-function evaluation budget. Perturbed elite positions are evaluated in the next regular population evaluation, so they do not add extra evaluations.

Table IV summarizes the average ranks and +/-/= results (see Table XXII in Appendix D). IWO achieves the best average rank, and each ablated variant obtains a worse overall rank. These results support the combined design of the five improvement strategies.

TABLE II
+/-/= COMPARISON SUMMARY FOR THE CEC2017 BENCHMARK FUNCTIONS

IWO vs	WO	WOA	DE	HOA	SGA	PSO	GWO	HHO
CEC2017 (D=30)	21/0/8	29/0/0	20/4/5	29/0/0	28/0/1	14/8/7	13/4/12	29/0/0
CEC2017 (D=50)	26/0/3	29/0/0	23/2/4	29/0/0	29/0/0	13/10/6	15/4/10	29/0/0
CEC2017 (D=100)	28/0/1	29/0/0	23/5/1	28/1/0	29/0/0	14/8/7	17/4/8	28/0/1
Total	75/0/12	87/0/0	66/11/10	86/1/0	86/0/1	41/26/20	45/12/30	86/0/1

TABLE III
SUMMARY OF THE RESULTS ON THE CONSTRAINED ENGINEERING BENCHMARK PROBLEMS

Problem	IWO mean objective	IWO rank	Best algorithm
Three-bar truss	2.6390e+02	1	IWO
Tension/compression spring	1.3691e-02	2	GWO
Speed reducer	2.9944e+03	1	IWO
Car side impact	2.2843e+01	1	IWO

TABLE IV
OVERALL RANKING AND +/-/= SUMMARY OF THE ABLATION STUDY

Metric	WO	IWO-noS1	IWO-noS2	IWO-noS3	IWO-noS4	IWO-noS5	IWO
Average Rank	6.90	4.48	2.90	4.66	3.69	3.66	1.72
+/-/=	0/29/0	3/26/0	6/23/0	3/26/0	5/24/0	4/25/0	-

IV. IWO-FMD FRAMEWORK

IWO is integrated with FMD to optimize the decomposition parameters for bearing fault feature extraction. In the proposed framework, each candidate position is mapped to a feasible FMD parameter combination and evaluated using the FCP-based fitness function. The selected parameters are then used for FMD decomposition and fault-sensitive mode extraction.

A. FMD processing and mode selection

In this study, FMD serves as the decomposition module, and IWO is used for parameter selection. Fig. 1 illustrates the FMD processing and mode-selection procedure using a simulated vibration signal. Figs. 1(a)–1(d) show the original signal and decomposed modes in the time and frequency domains, while Fig. 1(e) presents the corresponding envelope spectra. The mode with the largest envelope-spectrum response within the target-frequency band is selected as the fault-sensitive mode. In this example, Mode 1 exhibits the strongest response within this band. This example is provided only to explain the FMD processing procedure and is not used as quantitative evidence of IWO-FMD performance.

B. Parameter representation and constraint handling

Each IWO individual represents one candidate combination of four FMD parameters. Because FMD requires discrete and constrained parameter values, continuous IWO positions are converted into feasible integer parameter combinations through deterministic mapping and constraint repair.

1) *Search variables and ranges*: IWO-FMD jointly optimizes four FMD parameters: FilterSize, CutNum, ModeNum, and MaxIterNum. Therefore, the search dimension is $D=4$, and the four coordinates of each IWO candidate position correspond to these parameters, respectively.

FilterSize determines the adaptive filter length and affects the balance between time and frequency resolution. CutNum controls the initial partitioning of the frequency band, while ModeNum determines the number of retained feature modes after decomposition. MaxIterNum specifies the maximum number of FMD iterations and affects both convergence and computational cost. Table V lists the search ranges and constraints of these four parameters.

2) *Integer mapping and constraint repair*: Before each objective-function evaluation, the four continuous coordinates generated by IWO are rounded to the nearest integers and clipped to the ranges listed in Table V. If FilterSize is odd after rounding, it is adjusted by one to obtain a feasible even value. If $\text{CutNum} \leq \text{ModeNum}$, CutNum is set to $\text{ModeNum}+1$. Under the parameter ranges used in this study, the repaired values remain within their prescribed bounds.

The resulting feasible parameter combination is used for FMD decomposition and fitness evaluation. The mapping operation does not modify the continuous position stored by IWO. The calculated fitness value is assigned to the corresponding continuous candidate. After optimization terminates, the best continuous position is mapped again using the same procedure to obtain the final FMD parameter combination.

C. FCP-based fitness function

The fitness function is based on the envelope-spectrum response around the target fault characteristic frequency f_c .

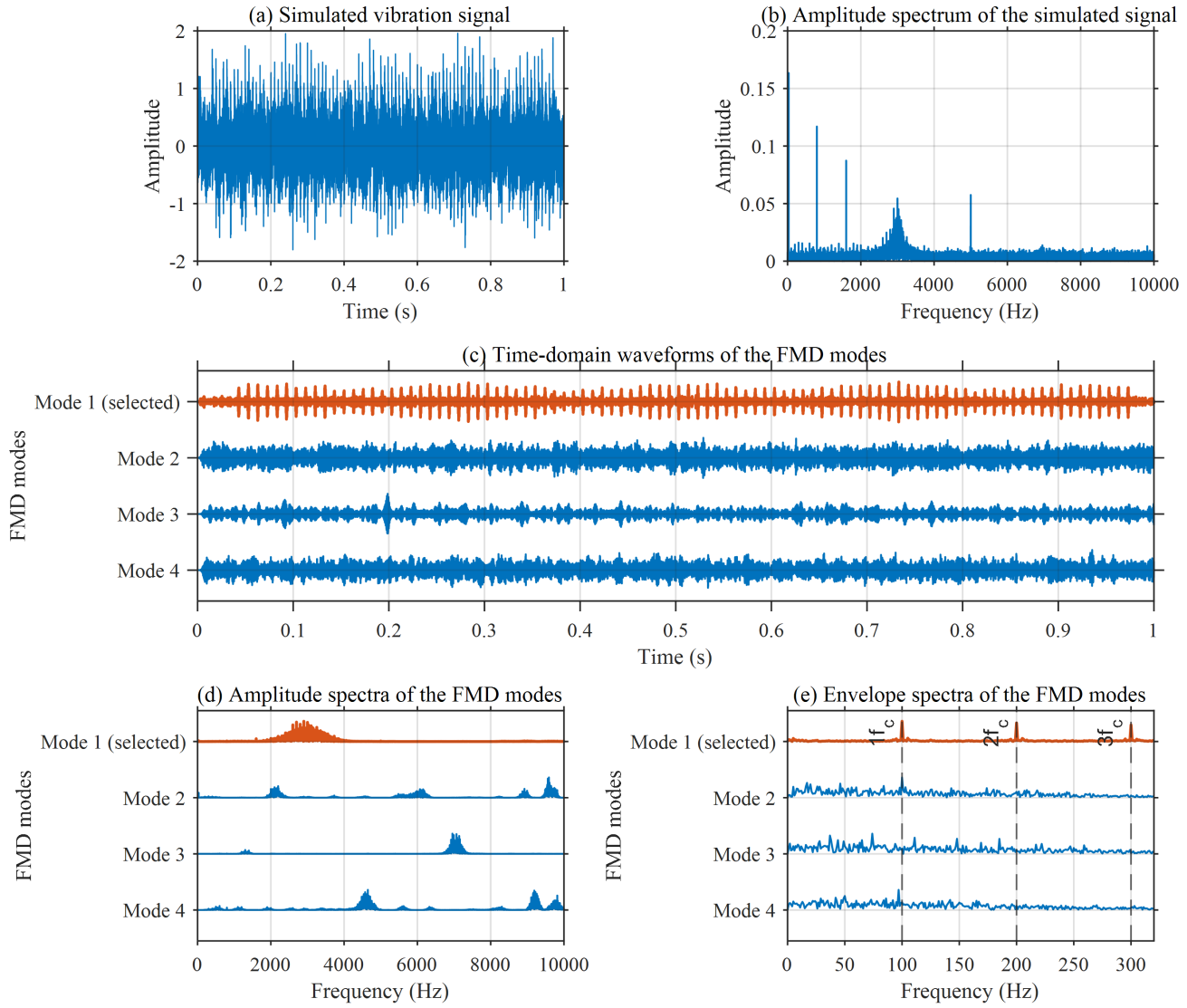


Fig. 1. Schematic of the FMD signal-processing procedure

TABLE V
SEARCH RANGES AND CONSTRAINTS OF FMD PARAMETERS USED IN IWO-FMD

FMD parameter	Search range	Type/constraint
FilterSize	[16, 64]	Even integer
CutNum	[8, 20]	Integer, CutNum>ModeNum
ModeNum	[2, 8]	Integer
MaxIterNum	[10, 50]	Integer

The value of f_c is determined by the fault type. For an outer race fault, f_c is the ball pass frequency of the outer race (BPFO). For an inner race fault, f_c is the ball pass frequency of the inner race (BPFI). For a rolling element fault, f_c is the ball spin frequency (BSF). These characteristic frequencies are calculated from the bearing geometry and shaft rotational frequency as follows:

$$f_{BPFO} = \frac{n_b}{2} f_r \left(1 - \frac{d_b}{D_p} \cos \theta \right) \quad (23)$$

$$f_{BPFI} = \frac{n_b}{2} f_r \left(1 + \frac{d_b}{D_p} \cos \theta \right) \quad (24)$$

$$f_{BSF} = \frac{D_p}{2d_b} f_r \left[1 - \left(\frac{d_b}{D_p} \cos \theta \right)^2 \right] \quad (25)$$

where n_b is the number of rolling elements, d_b is the rolling-element diameter, D_p is the pitch diameter, θ is the contact angle, and f_r is the shaft rotational frequency. For each vibration record, f_r is determined from the shaft speed under the

corresponding operating condition. Accordingly, $f_c = f_{\text{BPFO}}$ for an outer race fault, $f_c = f_{\text{BPFI}}$ for an inner race fault, and $f_c = f_{\text{BSF}}$ for a rolling element fault.

For each candidate parameter combination, FMD decomposes the search segment into $ModeNum$ modes. Let $A_h(f)$ denote the envelope-spectrum amplitude of the h -th mode at frequency f . The FCP of the h -th mode is defined as the maximum envelope-spectrum amplitude within a $\pm 5\%$ frequency band around f_c :

$$FCP_h = \max_{f \in [0.95f_c, 1.05f_c]} A_h(f), \quad h = 1, 2, \dots, ModeNum \quad (26)$$

where h is the mode index and $A_h(f)$ is the envelope-spectrum amplitude of the h -th mode at frequency f . The frequency band $[0.95f_c, 1.05f_c]$ accounts for finite frequency resolution and small deviations between the observed spectral peak and the theoretical characteristic frequency. A larger FCP_h indicates a stronger envelope-spectrum response of the h -th mode near f_c .

Because each candidate parameter combination produces multiple modes, the largest mode-level FCP is used to evaluate the current parameter combination:

$$FCP_{\max} = \max_{1 \leq h \leq ModeNum} FCP_h \quad (27)$$

where $FCP_{\max}$ denotes the largest FCP among all modes produced by the current candidate parameter combination. The mode that yields $FCP_{\max}$ is selected as the fault-sensitive mode. A larger $FCP_{\max}$ indicates that the current FMD parameter combination produces a stronger envelope-spectrum response near the target fault characteristic frequency.

Because IWO is formulated as a minimization algorithm, the fitness value of the current candidate parameter combination is defined as:

$$J = -FCP_{\max} \quad (28)$$

where J denotes the fitness value used by IWO. Therefore, maximizing $FCP_{\max}$ is equivalent to minimizing J .

The FCP-based fitness function is used only during parameter search. SNR and CK are calculated separately on the non-overlapping held-out evaluation segment and are not included in the fitness function. The target fault type and its characteristic frequency are assumed to be known before optimization.

D. IWO-FMD procedure

Fig. 2 illustrates the overall procedure of IWO-FMD. Each IWO candidate represents a combination of FilterSize, CutNum, ModeNum, and MaxIterNum. Before fitness evaluation, the continuous candidate position is converted into a feasible integer parameter combination according to Section IV-B2.

For each candidate, FMD is applied to the search segment, and the resulting modes are evaluated using the FCP-based fitness function defined in Section IV-C. The candidate population is then updated by IWO according to the strategies

described in Section II. The number of true objective-function evaluations is recorded throughout the optimization. If the remaining objective-function evaluation budget is smaller than the population size, only the required number of candidates is evaluated in the final partial generation.

After optimization, the parameter combination with the best search-segment fitness is selected. The selected combination is then applied unchanged to the non-overlapping held-out evaluation segment. FCP, SNR, and CK are then calculated on the held-out evaluation segment.

V. EXPERIMENTAL VALIDATION ON BEARING FAULT SIGNALS

This section evaluates IWO-FMD using vibration signals from the CWRU bearing dataset. The experiments evaluate bearing fault feature enhancement when the target fault characteristic frequency is known. Performance is assessed on non-overlapping held-out evaluation segments. The experimental setup, comparison methods, evaluation indicators, statistical analysis, and calibrated false-alarm and fault-detection analysis are described below.

A. Experimental setup

1) *Dataset and signal segmentation*: Fig. 3 shows the CWRU test platform, which includes a motor, torque transducer, dynamometer, and test bearings [37]. The experiments use drive-end vibration signals with 0.007 in seeded faults under four load conditions of 0, 1, 2, and 3 horsepower (HP). The considered fault types include inner race, outer race, and rolling element faults. The fault signals are sampled at 12 kHz. Normal baseline signals, used only for the calibrated false-alarm and fault-detection analysis, are resampled from 48 kHz to 12 kHz using anti-aliased resampling. Table VI summarizes the selected data settings.

The drive-end test bearing is a 6205-2RS JEM SKF bearing with nine rolling elements. The rolling-element diameter and pitch diameter are 0.3126 in and 1.537 in, respectively, and the contact angle is 0° . The approximate shaft speeds under 0, 1, 2, and 3 HP are 1797, 1772, 1750, and 1730 rpm, respectively.

Fault records are used in the main feature-enhancement experiments. Normal baseline records are used only for the calibrated false-alarm and fault-detection analysis. Each segment contains 2048 samples. Before FMD, each segment is reshaped into a column vector. No additional filtering or amplitude normalization is applied to the fault records.

For each repeated run, one 2048-point segment is used for parameter search. A separate, non-overlapping 2048-point segment from the same record is used for held-out evaluation. The parameter combination is selected using only the search segment. It is then applied unchanged to the corresponding held-out evaluation segment.

2) *Comparison methods and evaluation protocol*: IWO-FMD is first compared with fixed-parameter FMD. The fixed parameter combination is empirically set to [32, 12, 4, 25] for FilterSize, CutNum, ModeNum, and MaxIterNum, respectively, and is kept unchanged for all load-fault conditions.

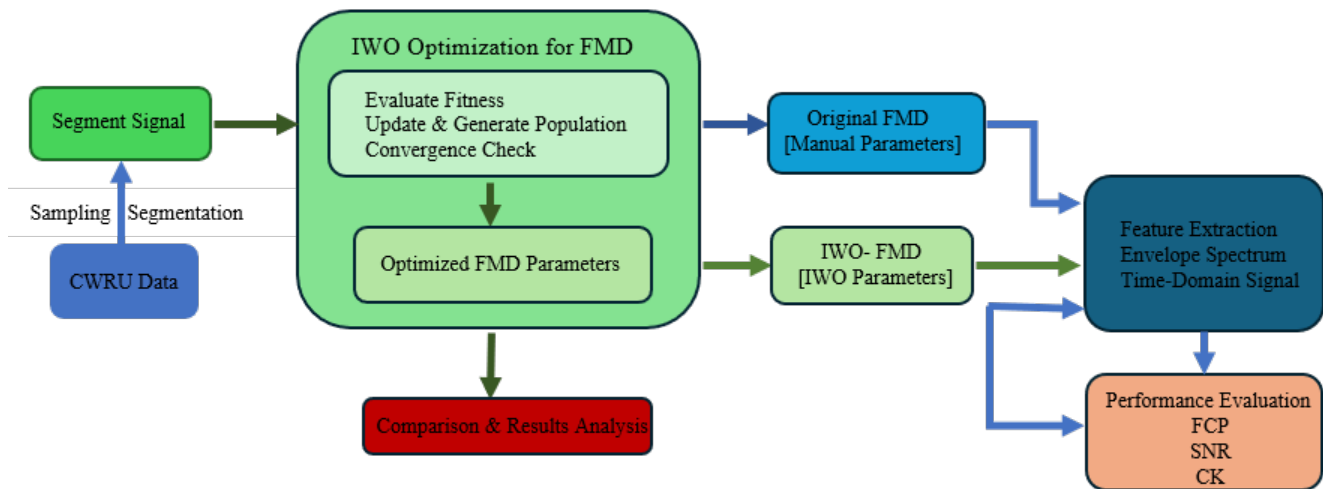


Fig. 2. Flowchart of the IWO-FMD framework

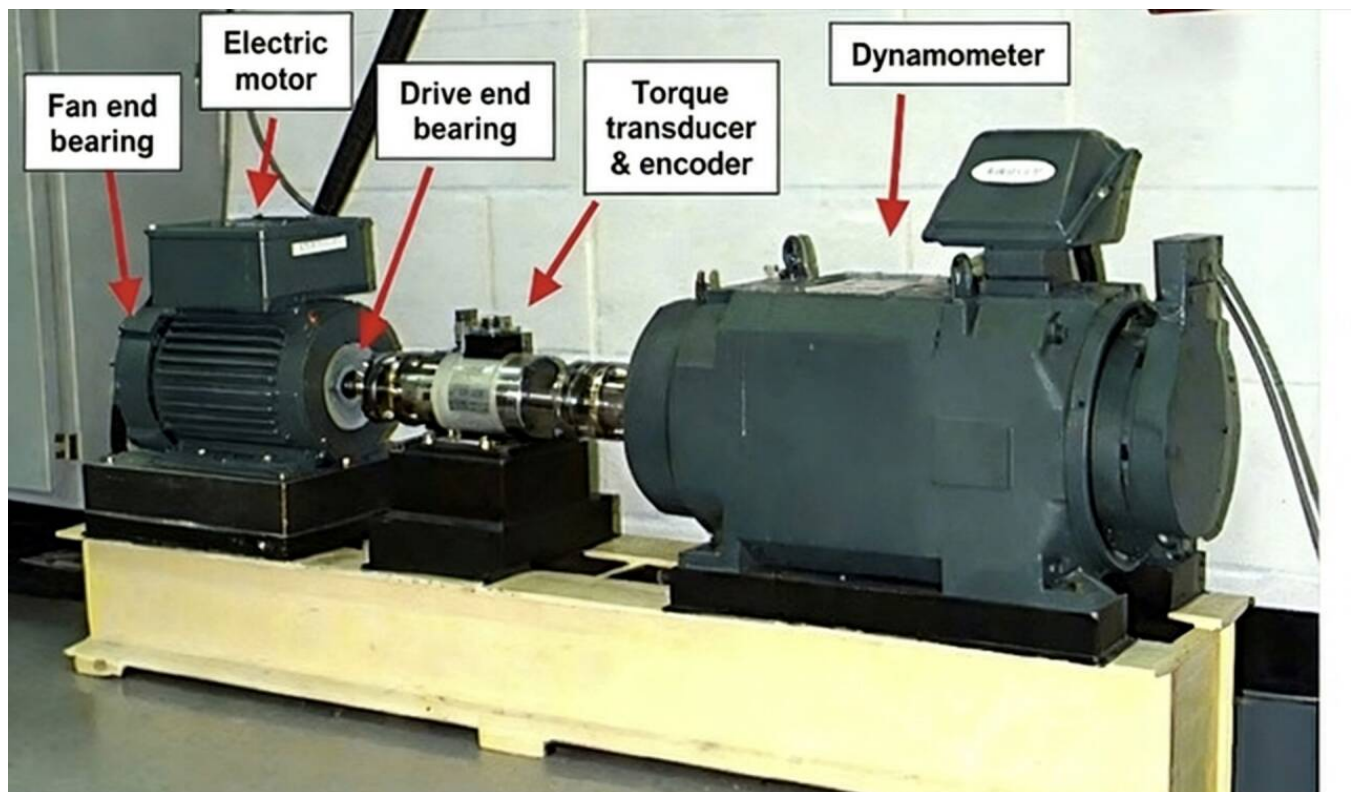


Fig. 3. Experimental platform of the CWRU bearing dataset

TABLE VI
DESCRIPTION OF THE SELECTED CWRU BEARING DATASET

Item	Setting
Dataset	CWRU bearing dataset
Sampling frequency	12 kHz fault data; normal data resampled from 48 to 12 kHz
Sensor position	Drive end
Fault size	0.007 in
Health condition	Normal, inner race fault, outer race fault, rolling element fault
Load condition	0, 1, 2, and 3 HP
Sample length	2048 points

A separate comparison includes WO-FMD, PSO-FMD, DE-FMD, HHO-FMD, and Octopus Optimization Algorithm-assisted FMD (OOA-FMD) [38]. All optimizer-assisted FMD methods use the parameter ranges listed in Table V and the same parameter mapping and constraint-repair rules.

After parameter search, the parameter combination with the best search-segment fitness is applied once to the corresponding non-overlapping held-out evaluation segment. This decomposition is used only for performance evaluation and is not included in the objective-function evaluation budget. The fixed-parameter FMD comparison and the optimizer-assisted FMD comparison are conducted as separate experimental blocks. Within each experimental block, all compared methods use the same search segments and held-out evaluation segments. Each load-fault condition is evaluated over ten repeated runs, and the results are reported as mean $\pm$ standard deviation.

3) *Evaluation indicators and statistical analysis*: FCP is the optimization response defined in Section IV-C. SNR and CK are used as auxiliary evaluation indicators for the fault-sensitive mode on the held-out evaluation segment. Neither SNR nor CK is involved in parameter search. For the fault-sensitive mode $s(n)$ and the original signal $x(n)$, the residual component is defined as $e(n)=x(n)-s(n)$. The SNR used in this study measures the energy ratio between the selected mode and the residual component. It is therefore different from the conventional SNR based on a known clean reference signal. The SNR is calculated as:

$$\text{SNR} = 10 \log_{10} \frac{\sum_{n=1}^{N_g} s^2(n)}{\sum_{n=1}^{N_g} e^2(n) + \varepsilon_{\text{SNR}}} \quad (29)$$

where N_g is the signal length and ε_{SNR} is a small positive constant used to avoid division by zero. In this study, $N_g = 2048$.

CK is used to evaluate the periodic impulsiveness of the fault-sensitive mode and is calculated as:

$$\text{CK} = \frac{\frac{1}{N_g - m\tau + 1} \sum_{n=m\tau}^{N_g} \left[\prod_{i=0}^{m-1} s(n - i\tau) \right]^2}{(\max |s(n)|^2 + \varepsilon_{\text{CK}})} \quad (30)$$

where $\tau = \text{round} \left(\frac{F_s}{f_c} \right)$ denotes the fault-period lag in samples, F_s is the sampling frequency, f_c is the theoretical fault characteristic frequency, and m is the shift order. In this study, $F_s = 12000$ Hz and $m = 4$. If $m\tau \geq N_g$, CK is assigned a very small value to avoid invalid calculation. These settings are kept the same for FMD and IWO-FMD. ε_{CK} is a small positive constant used to avoid division by zero.

Statistical significance is assessed only for SNR and CK in the comparison between fixed-parameter FMD and IWO-FMD. For each load-fault condition, ten paired held-out values are compared using a two-sided Wilcoxon signed-rank test at a significance level of 0.05. Since 20 tests are conducted for each indicator, the Benjamini-Hochberg procedure is applied separately to the SNR and CK tests to control the false discovery rate.

4) *Threshold calibration and fault detection*: The calibrated false-alarm and fault-detection analysis examines whether enhancing the response around a candidate fault characteristic frequency also increases false alarms in normal vibration signals. The CWRU normal baseline signals are resampled from 48 kHz to 12 kHz using anti-aliased resampling to match the sampling frequency of the fault signals.

For this calibrated analysis, three candidate-frequency conditions are considered. Nominal candidate fault characteristic frequencies of 157 Hz and 107 Hz are used for the inner race and outer race conditions, respectively. The corresponding $\pm 5\%$ frequency bands cover the load-specific theoretical BPFI and BPFO values. For the rolling element condition, the load-specific BSF values are 70.54, 69.60, 68.66, and 67.64 Hz under 0, 1, 2, and 3 HP, respectively.

For each load, the normal record is divided chronologically into a calibration pool and a test pool. The first half is used for threshold calibration, and the second half is used for false-alarm evaluation. Twenty calibration runs and ten test runs are conducted for each load. Each run contains one 2048-point search segment and one non-overlapping 2048-point held-out evaluation segment. For IWO-FMD, parameter search is performed only on the search segment, and the selected parameter combination is then applied unchanged to the corresponding held-out evaluation segment. For fixed-parameter FMD, the fixed parameter combination is applied directly to the held-out evaluation segment.

For each load, method, and candidate-frequency condition, the empirical 95th percentile of the calibration FCP values is used as the decision threshold. An evaluation FCP greater than or equal to the corresponding threshold is regarded as a false alarm for a normal segment and as a detection for a fault segment. For the BPFI and BSF conditions, each load contains ten matched fault runs. For the BPFO condition, ten runs are included for each of the three outer-race fault locations. The three locations therefore provide 30 fault runs per load. The same normal held-out evaluation segment is assessed under all three candidate-frequency conditions. Therefore, the resulting candidate-frequency decisions are not statistically independent.

B. Results and discussion

1) *Comparison with fixed-parameter FMD*: Tables VII–X report the held-out evaluation results under 0, 1, 2, and 3 HP, respectively. Each result is expressed as the mean $\pm$ standard deviation over ten repeated runs.

IWO-FMD achieves higher mean FCP and SNR values than fixed-parameter FMD in all 20 load-fault conditions. For CK, IWO-FMD achieves higher mean values in 19 of the 20 conditions. The magnitude of the improvement varies across fault types and load conditions.

The higher FCP values indicate that the parameter combinations selected by IWO-FMD strengthen the envelope-spectrum response within the target-frequency band. The SNR improvements indicate that the selected fault-sensitive modes generally contain more energy relative to the residual components. The CK results show that the improvement in periodic impulsiveness is also observed in most conditions. However, the CK

TABLE VII
HELD-OUT EVALUATION RESULTS AT 0.007 IN AND 0 HP

Fault condition	Quantitative index	FMD	IWO-FMD
Inner race fault	FCP	0.0162 $\pm$ 0.0136	0.1139 $\pm$ 0.0518
Inner race fault	SNR	-19.5159 $\pm$ 4.8199	-4.9456 $\pm$ 2.5764
Inner race fault	CK	3.2673e-08 $\pm$ 9.5612e-08	9.3114e-05 $\pm$ 1.4975e-04
Outer race fault – 3 o'clock position	FCP	0.0524 $\pm$ 0.0280	0.1493 $\pm$ 0.1002
Outer race fault – 3 o'clock position	SNR	-19.6180 $\pm$ 4.1092	-10.4260 $\pm$ 5.1869
Outer race fault – 3 o'clock position	CK	9.7911e-07 $\pm$ 1.7064e-06	0.0019 $\pm$ 0.0051
Outer race fault – 6 o'clock position	FCP	0.0256 $\pm$ 0.0074	0.1111 $\pm$ 0.0347
Outer race fault – 6 o'clock position	SNR	-22.8726 $\pm$ 2.4934	-11.1724 $\pm$ 1.8883
Outer race fault – 6 o'clock position	CK	2.9863e-06 $\pm$ 6.1086e-06	0.0018 $\pm$ 0.0032
Outer race fault – 12 o'clock position	FCP	0.0061 $\pm$ 0.0050	0.0242 $\pm$ 0.0134
Outer race fault – 12 o'clock position	SNR	-23.4951 $\pm$ 5.2054	-10.8693 $\pm$ 2.6622
Outer race fault – 12 o'clock position	CK	1.3706e-10 $\pm$ 4.1735e-10	1.0816e-07 $\pm$ 2.6308e-07
Rolling element fault	FCP	0.0032 $\pm$ 0.0027	0.0204 $\pm$ 0.0192
Rolling element fault	SNR	-22.3074 $\pm$ 9.2613	-8.5355 $\pm$ 6.9337
Rolling element fault	CK	1.6926e-11 $\pm$ 3.4868e-11	6.9540e-07 $\pm$ 1.5645e-06

TABLE VIII
HELD-OUT EVALUATION RESULTS AT 0.007 IN AND 1 HP

Fault condition	Quantitative index	FMD	IWO-FMD
Inner race fault	FCP	0.0235 $\pm$ 0.0119	0.1248 $\pm$ 0.0431
Inner race fault	SNR	-17.3372 $\pm$ 5.8632	-5.8459 $\pm$ 2.2927
Inner race fault	CK	5.7020e-08 $\pm$ 9.4610e-08	2.9280e-05 $\pm$ 2.1910e-05
Outer race fault – 3 o'clock position	FCP	0.0954 $\pm$ 0.1814	0.1102 $\pm$ 0.0876
Outer race fault – 3 o'clock position	SNR	-20.4910 $\pm$ 9.6409	-17.0813 $\pm$ 6.9746
Outer race fault – 3 o'clock position	CK	0.0044 $\pm$ 0.0139	0.0001 $\pm$ 0.0004
Outer race fault – 6 o'clock position	FCP	0.0293 $\pm$ 0.0131	0.1627 $\pm$ 0.0980
Outer race fault – 6 o'clock position	SNR	-23.4408 $\pm$ 3.4102	-10.8366 $\pm$ 4.6677
Outer race fault – 6 o'clock position	CK	6.2820e-08 $\pm$ 1.2580e-07	0.0082 $\pm$ 0.0244
Outer race fault – 12 o'clock position	FCP	0.0086 $\pm$ 0.0047	0.0384 $\pm$ 0.0317
Outer race fault – 12 o'clock position	SNR	-19.4080 $\pm$ 7.0084	-10.3897 $\pm$ 5.6321
Outer race fault – 12 o'clock position	CK	9.9000e-08 $\pm$ 3.1280e-07	2.0670e-06 $\pm$ 5.9550e-06
Rolling element fault	FCP	0.0051 $\pm$ 0.0037	0.0291 $\pm$ 0.0202
Rolling element fault	SNR	-16.3746 $\pm$ 6.5399	-6.6696 $\pm$ 8.1435
Rolling element fault	CK	7.3010e-10 $\pm$ 1.4400e-09	8.6190e-06 $\pm$ 2.2670e-05

TABLE IX
HELD-OUT EVALUATION RESULTS AT 0.007 IN AND 2 HP

Fault condition	Quantitative index	FMD	IWO-FMD
Inner race fault	FCP	0.0249 $\pm$ 0.0153	0.1225 $\pm$ 0.0754
Inner race fault	SNR	-17.2403 $\pm$ 7.0124	-7.6536 $\pm$ 3.8804
Inner race fault	CK	9.0890e-07 $\pm$ 2.1600e-06	0.0005 $\pm$ 0.0012
Outer race fault – 3 o'clock position	FCP	0.0446 $\pm$ 0.0468	0.1971 $\pm$ 0.1285
Outer race fault – 3 o'clock position	SNR	-22.7400 $\pm$ 8.1853	-13.7247 $\pm$ 7.7781
Outer race fault – 3 o'clock position	CK	5.0710e-06 $\pm$ 1.3180e-05	0.0010 $\pm$ 0.0015
Outer race fault – 6 o'clock position	FCP	0.0319 $\pm$ 0.0147	0.1578 $\pm$ 0.0571
Outer race fault – 6 o'clock position	SNR	-22.7722 $\pm$ 3.8296	-10.1106 $\pm$ 2.6045
Outer race fault – 6 o'clock position	CK	5.4230e-08 $\pm$ 1.0590e-07	0.0002 $\pm$ 0.0003
Outer race fault – 12 o'clock position	FCP	0.0213 $\pm$ 0.0179	0.0986 $\pm$ 0.0958
Outer race fault – 12 o'clock position	SNR	-18.3840 $\pm$ 8.3591	-8.8845 $\pm$ 6.0698
Outer race fault – 12 o'clock position	CK	1.1590e-08 $\pm$ 2.4230e-08	1.2750e-05 $\pm$ 2.8080e-05
Rolling element fault	FCP	0.0041 $\pm$ 0.0039	0.0258 $\pm$ 0.0180
Rolling element fault	SNR	-15.9377 $\pm$ 4.0639	-5.8084 $\pm$ 5.1284
Rolling element fault	CK	8.6360e-10 $\pm$ 2.6720e-09	2.7910e-06 $\pm$ 5.0740e-06

TABLE X
HELD-OUT EVALUATION RESULTS AT 0.007 IN AND 3 HP

Fault condition	Quantitative index	FMD	IWO-FMD
Inner race fault	FCP	0.0153 $\pm$ 0.0100	0.0658 $\pm$ 0.0134
Inner race fault	SNR	-21.6250 $\pm$ 5.3394	-8.5332 $\pm$ 1.9959
Inner race fault	CK	5.7680e-09 $\pm$ 1.4400e-08	3.9510e-06 $\pm$ 4.8940e-06
Outer race fault – 3 o'clock position	FCP	0.1261 $\pm$ 0.1540	0.2648 $\pm$ 0.2016
Outer race fault – 3 o'clock position	SNR	-18.3176 $\pm$ 9.1800	-10.4774 $\pm$ 9.3387
Outer race fault – 3 o'clock position	CK	0.0016 $\pm$ 0.0048	0.0164 $\pm$ 0.0421
Outer race fault – 6 o'clock position	FCP	0.0311 $\pm$ 0.0222	0.1954 $\pm$ 0.1175
Outer race fault – 6 o'clock position	SNR	-21.8064 $\pm$ 4.8887	-8.5099 $\pm$ 5.5217
Outer race fault – 6 o'clock position	CK	1.0150e-07 $\pm$ 2.5110e-07	0.0010 $\pm$ 0.0022
Outer race fault – 12 o'clock position	FCP	0.0081 $\pm$ 0.0059	0.0560 $\pm$ 0.0409
Outer race fault – 12 o'clock position	SNR	-20.7245 $\pm$ 8.6570	-8.3786 $\pm$ 5.3056
Outer race fault – 12 o'clock position	CK	3.7530e-07 $\pm$ 1.1860e-06	1.4110e-06 $\pm$ 2.6120e-06
Rolling element fault	FCP	0.0036 $\pm$ 0.0014	0.0228 $\pm$ 0.0099
Rolling element fault	SNR	-19.4154 $\pm$ 5.9310	-5.7392 $\pm$ 3.2503
Rolling element fault	CK	2.2980e-10 $\pm$ 5.6970e-10	6.1150e-07 $\pm$ 1.0650e-06

improvement is not obtained in every condition, indicating that enhancement of the target-frequency response does not necessarily improve all impulsiveness-related characteristics simultaneously.

Because SNR and CK are calculated on held-out evaluation segments, their improvements indicate that the selected parameter combinations remain effective beyond the search segments. These results support bearing fault feature enhancement under the tested conditions but should not be interpreted as evidence of universal noise suppression or complete fault-diagnosis capability. Table XI reports the paired Wilcoxon signed-rank test results for SNR and CK.

Note: Median Δ SNR and Median Δ CK are calculated as median(IWO-FMD - FMD) over ten paired held-out runs. The adjusted p-values are obtained from a two-sided paired Wilcoxon signed-rank test followed by Benjamini–Hochberg false discovery rate correction. “Yes” indicates $p_{\text{adj}} < 0.05$. FCP is not tested here because it is directly used as the optimization response in the fitness function.

A positive median difference indicates a larger value for IWO-FMD. After Benjamini–Hochberg correction, the SNR improvement is statistically significant in 19 of the 20 conditions, whereas the CK improvement is significant in 17 conditions. The lower consistency of CK indicates that strengthening the target-frequency response does not always produce a corresponding increase in periodic impulsiveness.

Figs. 4 and 5 present representative fault-sensitive-mode waveforms and envelope spectra under 0 and 1 HP, respectively. For each load–fault condition, the displayed run is selected as the run whose paired SNR improvement is closest to the median value of the ten repeated runs. This selection avoids using only the most favorable case. Compared with fixed-parameter FMD, IWO-FMD generally produces stronger envelope-spectrum responses near the fault characteristic frequency. Stronger harmonic components are also observed in some conditions. These figures provide representative visual examples under 0 and 1 HP, consistent with the quantitative trends reported in Tables VII–XI.

2) Comparison with optimizer-assisted FMD methods:

Table XII summarizes the descriptive win/tie/loss results of

IWO-FMD against WO-FMD, PSO-FMD, DE-FMD, HHO-FMD, and OOA-FMD across the 20 load–fault conditions. The comparison is based on the mean held-out FCP, SNR, and CK values over ten repeated runs for each condition. Detailed results are provided (see Tables XXIII–XXVI in Appendix E).

Note: W/T/L denotes the numbers of load–fault conditions in which the mean held-out value of IWO-FMD is higher than, equal to, or lower than that of the corresponding baseline, respectively. The W/T/L results are descriptive comparisons and do not represent statistical significance.

IWO-FMD records more wins than losses for both FCP and CK against all five optimizer-assisted FMD methods. For FCP, the win/loss counts range from 12/8 against PSO-FMD and HHO-FMD to 18/2 against OOA-FMD. The SNR results are less consistent. IWO-FMD records more wins than losses against WO-FMD and OOA-FMD, equal numbers of wins and losses against PSO-FMD and DE-FMD, and fewer wins than losses against HHO-FMD. These results indicate that the relative performance of IWO-FMD is more consistent for FCP and CK than for SNR. Overall, IWO-FMD remains competitive with the five optimizer-assisted FMD methods under the same objective-function evaluation budget. However, the descriptive W/T/L results do not indicate that IWO-FMD outperforms every comparison method under every load–fault condition.

3) *Calibrated false-alarm and fault-detection results:* Table XIII reports the calibrated false-alarm and fault-detection results. The decision threshold is determined separately for each load, method, and candidate-frequency condition using the corresponding calibration data described in Section V-A4. Therefore, each method is evaluated relative to its own normal-response distribution rather than using a common FCP threshold.

The false-alarm results are interpreted descriptively because each load–frequency condition contains only ten normal test runs. In addition, the same normal held-out evaluation segment is assessed under the three candidate-frequency conditions. The resulting candidate-frequency decisions are therefore not statistically independent.

Across all fault runs, the pooled fault-detection rate in-

TABLE XI
STATISTICAL SIGNIFICANCE TEST RESULTS FOR AUXILIARY EVALUATION INDICATORS

Load	Fault condition	Median Δ SNR	p_adj(SNR)	Sig.(SNR)	Median Δ CK	p_adj(CK)	Sig.(CK)
0HP	Inner race fault	16.3755	0.0040	Yes	2.9655e-05	0.0040	Yes
0HP	Outer race - 3 o'clock	10.0057	0.0040	Yes	6.4879e-05	0.0040	Yes
0HP	Outer race - 6 o'clock	11.5288	0.0040	Yes	7.2930e-04	0.0040	Yes
0HP	Outer race - 12 o'clock	13.2772	0.0060	Yes	1.7835e-08	0.0060	Yes
0HP	Rolling element fault	12.0881	0.0040	Yes	2.7314e-08	0.0040	Yes
1HP	Inner race fault	9.3225	0.0040	Yes	2.5460e-05	0.0040	Yes
1HP	Outer race - 3 o'clock	2.7749	0.3750	No	4.8908e-07	0.3750	No
1HP	Outer race - 6 o'clock	14.3645	0.0060	Yes	4.9260e-04	0.0060	Yes
1HP	Outer race - 12 o'clock	8.3524	0.0300	Yes	3.8756e-08	0.0680	No
1HP	Rolling element fault	11.3944	0.0170	Yes	5.3616e-07	0.0080	Yes
2HP	Inner race fault	10.1669	0.0040	Yes	3.1512e-05	0.0060	Yes
2HP	Outer race - 3 o'clock	11.2886	0.0300	Yes	4.0110e-04	0.0230	Yes
2HP	Outer race - 6 o'clock	13.7455	0.0040	Yes	6.7109e-05	0.0040	Yes
2HP	Outer race - 12 o'clock	9.1520	0.0130	Yes	4.5143e-08	0.0170	Yes
2HP	Rolling element fault	10.1755	0.0040	Yes	4.6086e-07	0.0040	Yes
3HP	Inner race fault	13.2852	0.0040	Yes	2.0788e-06	0.0040	Yes
3HP	Outer race - 3 o'clock	7.6958	0.0040	Yes	1.9813e-04	0.0060	Yes
3HP	Outer race - 6 o'clock	11.9015	0.0040	Yes	2.7002e-05	0.0040	Yes
3HP	Outer race - 12 o'clock	14.1597	0.0080	Yes	3.8209e-07	0.0680	No
3HP	Rolling element fault	12.1133	0.0040	Yes	1.6644e-07	0.0040	Yes

TABLE XII
DESCRIPTIVE WIN/TIE/LOSS COMPARISON OF IWO-FMD AGAINST OPTIMIZER-ASSISTED FMD BASELINES

Comparison	FCP W/T/L	SNR W/T/L	CK W/T/L
IWO-FMD vs WO-FMD	14/0/6	13/0/7	15/0/5
IWO-FMD vs PSO-FMD	12/0/8	10/0/10	13/0/7
IWO-FMD vs DE-FMD	15/0/5	10/0/10	15/0/5
IWO-FMD vs HHO-FMD	12/0/8	9/0/11	14/0/6
IWO-FMD vs OOA-FMD	18/0/2	18/0/2	17/0/3

creases from 45.0% (90/200) for fixed-parameter FMD to 60.5% (121/200) for IWO-FMD. Because three outer-race fault locations are included, outer-race faults contribute more runs to the pooled result. The detection rates are therefore also examined by fault type. For inner-race, outer-race, and rolling-element faults, the rates increase from 65.0% to 95.0%, from 49.2% to 62.5%, and from 12.5% to 20.0%, respectively.

For normal signals, the pooled candidate-level false-alarm rate decreases from 10.8% (13/120) to 7.5% (9/120). The false-alarm rates of fixed-parameter FMD and IWO-FMD are 12.5% and 12.5% for BPFI, 10.0% and 7.5% for BPFO, and 10.0% and 2.5% for BSF, respectively.

The improvement is not observed in every load-fault condition. IWO-FMD produces lower detection rates in the 1 HP and 2 HP outer-race conditions and in the 3 HP rolling-element condition. Both methods produce a 0% detection

rate in the 1 HP rolling-element condition. IWO-FMD also produces higher false-alarm rates in the 0 HP outer-race, 2 HP outer-race, and 3 HP inner-race conditions. Overall, the calibrated analysis shows a more favorable pooled balance for IWO-FMD. However, these results should be interpreted as descriptive evidence under the tested conditions.

VI. CONCLUSIONS AND FUTURE WORK

This study proposes an IWO-FMD framework for adaptive FMD parameter selection under approximately fixed-speed conditions with known fault characteristic frequencies. IWO-FMD jointly optimizes FilterSize, CutNum, ModeNum, and MaxIterNum using FCP as the optimization response. Parameter search and held-out evaluation use separate, non-overlapping signal segments.



Fig. 4. Comparison of envelope spectra and fault-sensitive modes under 0 HP load

TABLE XIII
CALIBRATED FALSE-ALARM AND FAULT-DETECTION RESULTS

Load	Candidate fault characteristic frequency	FMD threshold	FMD FAR	FMD detection rate	IWO-FMD threshold	IWO-FMD FAR	IWO-FMD detection rate
0 HP	Nominal BPFI (157.00 Hz)	0.015764	10.0% (1/10)	40.0% (4/10)	0.029093	0.0% (0/10)	90.0% (9/10)
0 HP	Nominal BPFO (107.00 Hz)	0.019659	10.0% (1/10)	43.3% (13/30)	0.025609	20.0% (2/10)	80.0% (24/30)
0 HP	BSF (70.54 Hz)	0.020557	0.0% (0/10)	0.0% (0/10)	0.050411	0.0% (0/10)	20.0% (2/10)
1 HP	Nominal BPFI (157.00 Hz)	0.013204	0.0% (0/10)	70.0% (7/10)	0.028390	0.0% (0/10)	100.0% (10/10)
1 HP	Nominal BPFO (107.00 Hz)	0.013978	20.0% (2/10)	63.3% (19/30)	0.059488	0.0% (0/10)	56.7% (17/30)
1 HP	BSF (69.60 Hz)	0.012630	10.0% (1/10)	0.0% (0/10)	0.037280	0.0% (0/10)	0.0% (0/10)
2 HP	Nominal BPFI (157.00 Hz)	0.009397	10.0% (1/10)	80.0% (8/10)	0.026378	10.0% (1/10)	100.0% (10/10)
2 HP	Nominal BPFO (107.00 Hz)	0.010168	0.0% (0/10)	80.0% (24/30)	0.039059	10.0% (1/10)	73.3% (22/30)
2 HP	BSF (68.66 Hz)	0.007331	10.0% (1/10)	0.0% (0/10)	0.028411	0.0% (0/10)	40.0% (4/10)
3 HP	Nominal BPFI (157.00 Hz)	0.005289	30.0% (3/10)	70.0% (7/10)	0.019987	40.0% (4/10)	90.0% (9/10)
3 HP	Nominal BPFO (107.00 Hz)	0.036359	10.0% (1/10)	10.0% (3/30)	0.096138	0.0% (0/10)	40.0% (12/30)
3 HP	BSF (67.64 Hz)	0.005903	20.0% (2/10)	50.0% (5/10)	0.027387	10.0% (1/10)	20.0% (2/10)

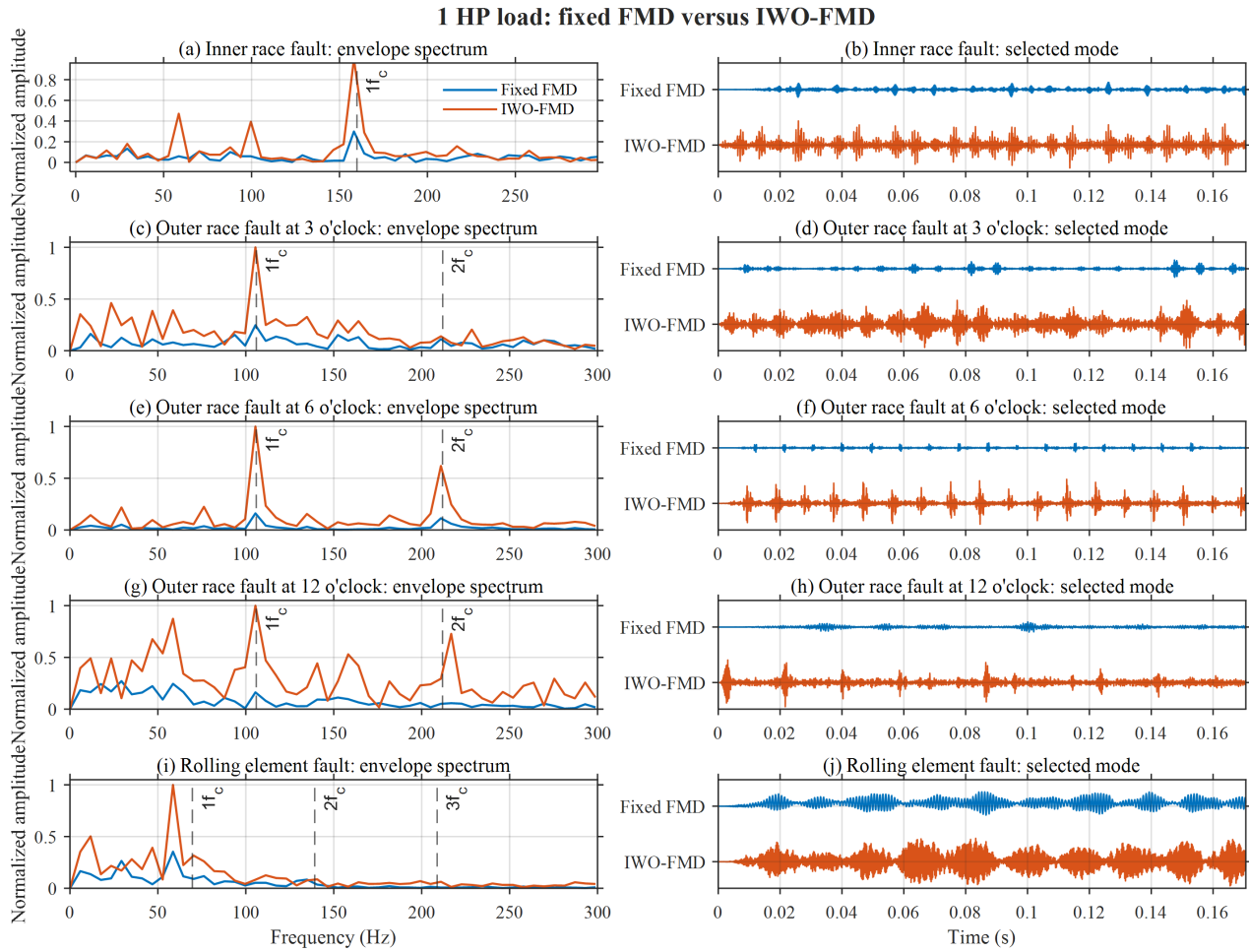


Fig. 5. Comparison of envelope spectra and fault-sensitive modes under 1 HP load

On the CWRU bearing dataset, IWO-FMD achieves higher mean FCP and SNR values than fixed-parameter FMD in all 20 tested load–fault conditions. It also achieves higher mean CK values in 19 conditions. After Benjamini–Hochberg correction, the SNR and CK improvements are statistically significant in 19 and 17 conditions, respectively. Under the same budget of 100 true objective-function evaluations, IWO-FMD records more wins than losses for FCP and CK against all five optimizer-assisted FMD methods. In the calibrated analysis, the pooled fault-detection rate increases from 45.0% to 60.5%, while the candidate-level false-alarm rate decreases from 10.8% to 7.5%. These results indicate that IWO-FMD improves FMD parameter selection under the tested conditions. It also achieves a better balance between bearing fault feature enhancement and false alarms.

The proposed framework still has several limitations. It assumes approximately fixed-speed conditions and prior knowledge of the target fault characteristic frequency. The experiments are also limited to the CWRU dataset with seeded bearing faults, and improvements are not observed in every tested load–fault condition. Future work will investigate naturally developed faults, additional bearing datasets, variable-speed conditions, adaptive fault-frequency tracking, computational

acceleration, and more comprehensive decision-threshold calibration.

STATEMENTS AND DECLARATIONS

Competing interests The authors report no competing financial interests or personal relationships that could have influenced this work.

Author contributions Omitted for anonymous review.

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Data availability The CWRU bearing dataset is public. The data-file identifiers, random seeds, search and evaluation segment indices, selected FMD parameters, evaluation counts, processed data, and raw results are available from the corresponding author upon reasonable request.

APPENDIX A

Detailed update equations of WO

This appendix presents the original WO position-update equations used in IWO, including migration, roosting, fleeing, and gathering behaviors.

Migration (exploration)

During migration, each walrus moves according to the position difference between two randomly selected sentinel walruses:

$$X_{i,j}^{t+1} = X_{i,j}^t + \text{Migration_step} \quad (\text{A1})$$

$$\text{Migration_step} = \beta \cdot (X_{m,j}^t - X_{n,j}^t) \cdot (\text{rand}_6)^2 \quad (\text{A2})$$

$$\beta = 1 - \frac{1}{1 + \exp\left(-\frac{t - \frac{\text{MaxIter}}{2}}{\frac{\text{MaxIter}}{2}} \cdot 10\right)} \quad (\text{A3})$$

where $X_{i,j}^t$ and $X_{i,j}^{t+1}$ denote the current and updated values of the j -th position component of the i -th walrus at iteration t , respectively. Migration_step is the migration step vector. $X_{m,j}^t$ and $X_{n,j}^t$ denote the j -th position components of two different sentinel walruses randomly selected from the current population, where $m \neq n$. β is a decay factor used to control the migration step size. rand_6 denotes a random vector uniformly distributed in $(0, 1)$.

Reproduction (exploitation)**(1) Roosting behavior**

During roosting, male walruses are redistributed within the search bounds using a Halton sequence. Female walruses are guided by the corresponding male walruses and the current best position, whereas juvenile walruses update their positions using a reference safe position and Lévy flight.

$$\begin{aligned} \text{Female}_{i,j}^{t+1} &= \text{Female}_{i,j}^t + \alpha \cdot (\text{Male}_{i,j}^t - \text{Female}_{i,j}^t) \\ &+ (1 - \alpha) \cdot (X_{\text{best},j}^t - \text{Female}_{i,j}^t) \end{aligned} \quad (\text{A4})$$

$$\text{Juvenile}_{i,j}^{t+1} = (O_{i,j}^t - \text{Juvenile}_{i,j}^t) \cdot \text{rand}_7 \quad (\text{A5})$$

$$O_{i,j}^t = X_{\text{best},j}^t + \text{Juvenile}_{i,j}^t \cdot LF \quad (\text{A6})$$

$$LF = 0.05 \cdot \frac{u}{|v|^{\frac{1}{\lambda}}} \quad (\text{A7})$$

$$u \sim N(0, \sigma_u^2), v \sim N(0, \sigma_v^2) \quad (\text{A8})$$

$$\sigma_u = \left[\frac{\Gamma(1 + \lambda) \cdot \sin(\frac{\pi\lambda}{2})}{\Gamma(\frac{1+\lambda}{2}) \cdot \lambda \cdot 2^{\frac{\lambda-1}{2}}} \right]^{\frac{1}{\lambda}}, \sigma_v = 1, \lambda = 1.5 \quad (\text{A9})$$

where $\text{Female}_{i,j}^t$ and $\text{Female}_{i,j}^{t+1}$ denote the current and updated position components of the i -th female walrus in the j -th dimension, respectively. $\text{Male}_{i,j}^t$ is the corresponding

male position component, and $X_{\text{best},j}^t$ is the current global best position vector. α is defined in Eq. (4). $\text{Juvenile}_{i,j}^t$ and $\text{Juvenile}_{i,j}^{t+1}$ denote the current and updated juvenile position components. $O_{i,j}^t$ is the reference safe position vector of the i -th juvenile walrus in the j -th dimension. rand_7 is a random number uniformly distributed in $(0, 1)$. LF is a Lévy flight vector generated from the normally distributed random vectors u and v . σ_u and σ_v are their standard deviations, λ is the Lévy distribution parameter, and $\Gamma(\cdot)$ denotes the Gamma function.

(2) Foraging behavior

Foraging consists of fleeing and gathering.

(a) Fleeing behavior

During fleeing, each walrus updates its position using the current best position and a random disturbance factor:

$$\begin{aligned} X_{i,j}^{t+1} &= X_{i,j}^t \cdot R \\ &- |X_{\text{best},j}^t - X_{i,j}^t| \cdot (\text{rand}_8)^2 \end{aligned} \quad (\text{A10})$$

where R is the random disturbance factor defined in Eq. (6), and rand_8 is a random number in $(0, 1)$.

(b) Gathering behavior

During gathering, WO generates two temporary candidate values for each position component and combines them with equal weights.

$$X_{i,j}^{t+1} = \frac{X_1 + X_2}{2} \quad (\text{A11})$$

$$\begin{aligned} X_1 &= X_{\text{best},j}^t - a_1 \cdot b_1 \\ &\cdot |X_{\text{best},j}^t - X_{i,j}^t| \end{aligned} \quad (\text{A12})$$

$$\begin{aligned} X_2 &= X_{\text{second},j}^t - a_2 \cdot b_2 \\ &\cdot |X_{\text{second},j}^t - X_{i,j}^t| \end{aligned} \quad (\text{A13})$$

$$\begin{aligned} a_1 &= \beta \cdot \text{rand}_9 - \beta \\ , \quad a_2 &= \beta \cdot \text{rand}_{10} - \beta \end{aligned} \quad (\text{A14})$$

$$b_1 = \tan(\theta_1), \quad b_2 = \tan(\theta_2) \quad (\text{A15})$$

where X_1 and X_2 are two temporary candidate values generated for this position component using the current best and second best positions, respectively. $X_{\text{second},j}^t$ represents the second best position vector. a_1 , a_2 , b_1 , and b_2 are gathering coefficients. rand_9 and rand_{10} are random numbers uniformly distributed in $(0, 1)$, and θ_1 and θ_2 are random angles in $(0, \pi)$.

TABLE XIV
CEC2017 RESULTS AT 30 DIMENSIONS

Function	Index	IWO	WO	WOA	DE	HOA	SGA	PSO	GWO	HHO
F1	Best	1.6983e+05	3.7686e+07	2.0871e+09	2.0619e+05	2.4587e+10	1.6903e+09	1.5273e+06	4.6285e+08	7.6207e+09
F1	Mean	1.1941e+06	1.3357e+08	4.9273e+09	3.9859e+05	3.5430e+10	3.6484e+09	3.1580e+06	3.0205e+09	1.7267e+10
F1	Std	8.1898e+05	6.5206e+07	1.9675e+09	1.2639e+05	5.7790e+09	2.0051e+09	1.5181e+06	2.1445e+09	4.1549e+09
F3	Best	1.9401e+04	4.6467e+04	1.3648e+05	8.2551e+04	5.0668e+04	4.7052e+04	2.0755e+04	2.9874e+04	6.8337e+04
F3	Mean	4.1399e+04	6.5543e+04	2.6103e+05	1.2646e+05	7.1354e+04	6.4036e+04	4.5770e+04	5.9101e+04	8.0138e+04
F3	Std	9.6694e+03	7.7689e+03	7.1724e+04	1.9637e+04	8.0424e+03	9.5496e+03	1.5798e+04	1.4862e+04	5.5838e+03
F4	Best	4.8903e+02	4.9522e+02	8.6872e+02	5.3244e+02	4.9595e+03	6.1887e+02	4.0589e+02	5.0717e+02	1.3110e+03
F4	Mean	5.3337e+02	5.7064e+02	1.4170e+03	5.5958e+02	7.4897e+03	8.6806e+02	5.0342e+02	6.8786e+02	3.3881e+03
F4	Std	3.1151e+01	5.2210e+01	3.7522e+02	1.3484e+01	1.6357e+03	2.1012e+02	3.1471e+01	3.3950e+02	8.8232e+02
F5	Best	5.5004e+02	5.8564e+02	6.9041e+02	6.3177e+02	7.4817e+02	6.6810e+02	6.1795e+02	5.6947e+02	7.6454e+02
F5	Mean	6.2546e+02	6.7628e+02	8.6570e+02	6.5938e+02	8.1289e+02	7.8530e+02	6.8328e+02	6.4572e+02	8.4152e+02
F5	Std	6.5949e+01	7.7979e+01	7.4357e+01	1.1999e+01	3.6337e+01	5.9907e+01	2.8288e+01	4.2692e+01	2.9856e+01
F6	Best	6.0663e+02	6.1145e+02	6.6147e+02	6.0016e+02	6.5211e+02	6.4841e+02	6.4147e+02	6.0436e+02	6.6345e+02
F6	Mean	6.2071e+02	6.3520e+02	6.8539e+02	6.0026e+02	6.6653e+02	6.6803e+02	6.5142e+02	6.1328e+02	6.8015e+02
F6	Std	7.5470e+00	1.6391e+01	1.1194e+01	6.0955e+02	7.0658e+00	9.9916e+00	6.9034e+00	5.0339e+00	6.6833e+00
F7	Best	7.9816e+02	8.6012e+02	1.1010e+03	8.5549e+02	1.1136e+03	1.0585e+03	8.5731e+02	8.2327e+02	1.2040e+03
F7	Mean	8.7164e+02	9.7979e+02	1.2873e+03	9.0276e+02	1.2437e+03	1.2425e+03	9.3528e+02	8.9703e+02	1.3731e+03
F7	Std	6.6012e+01	7.7737e+01	8.6675e+01	1.8122e+01	8.2956e+01	9.7114e+01	4.1515e+01	5.5870e+01	6.2639e+01
F8	Best	8.6379e+02	8.7621e+02	9.9684e+02	9.5546e+02	1.0005e+03	9.6547e+02	8.7670e+02	8.4765e+02	1.0092e+03
F8	Mean	9.1138e+02	9.6094e+02	1.0746e+03	9.6858e+02	1.0648e+03	1.0166e+03	9.3117e+02	9.3127e+02	1.0555e+03
F8	Std	3.4908e+01	7.0558e+01	5.0192e+01	8.4028e+00	2.5672e+01	3.1388e+01	2.4989e+01	5.6034e+01	3.2160e+01
F9	Best	1.1749e+03	1.8266e+03	6.7005e+03	1.2886e+03	3.3802e+03	3.6213e+03	3.3724e+03	7.7585e+03	7.585e+03
F9	Mean	3.8716e+03	5.4682e+03	1.1117e+04	1.9072e+03	6.9363e+03	7.3532e+03	6.3160e+03	2.3625e+03	9.9372e+03
F9	Std	2.6223e+03	2.4083e+03	3.2750e+03	2.3779e+02	1.4595e+03	1.7233e+03	1.6182e+03	7.4175e+02	1.3249e+03
F10	Best	3.5334e+03	4.2231e+03	5.5332e+03	6.4534e+03	5.3352e+03	4.7286e+03	3.7392e+03	3.6637e+03	6.4314e+03
F10	Mean	6.8252e+03	8.3371e+03	7.4512e+03	6.9152e+03	7.4286e+03	6.2503e+03	4.9267e+03	5.0625e+03	7.7349e+03
F10	Std	2.2453e+03	1.4053e+03	7.7789e+02	2.7537e+02	6.3810e+02	5.7793e+02	6.6908e+02	1.3038e+03	7.1539e+02
F11	Best	1.1435e+03	1.2751e+03	6.5930e+03	1.3772e+03	3.5074e+03	1.5377e+03	1.1545e+03	1.3742e+03	2.4933e+03
F11	Mean	1.2927e+03	1.3928e+03	1.1144e+04	2.1901e+03	5.9889e+03	2.0073e+03	1.2297e+03	2.1800e+03	6.9303e+03
F11	Std	7.1717e+01	1.1130e+02	3.3013e+02	5.2347e+02	1.8118e+02	4.2449e+02	4.9049e+01	9.9408e+02	1.9181e+03
F12	Best	3.5981e+05	4.6218e+06	8.2912e+07	4.8058e+06	2.8309e+07	1.8809e+07	4.5068e+05	6.5906e+06	5.7789e+08
F12	Mean	1.3431e+07	3.5842e+07	5.0705e+08	1.2629e+07	6.1505e+09	1.7289e+08	2.7256e+06	7.8170e+07	2.0767e+09
F12	Std	1.2564e+07	2.5287e+07	5.0705e+08	6.0002e+06	1.9622e+09	1.0488e+08	2.3066e+06	8.8338e+07	1.0111e+09
F13	Best	2.5474e+04	4.5147e+04	6.3754e+05	2.7018e+06	3.5512e+08	8.460e+04	6.7925e+05	5.0838e+04	8.5733e+06
F13	Mean	1.5703e+05	2.4543e+05	1.4174e+07	2.7018e+06	2.6613e+09	1.1896e+06	2.6833e+04	1.7675e+07	2.5212e+08
F13	Std	1.5104e+05	1.8091e+05	1.4629e+07	1.9339e+06	1.6513e+09	2.1852e+06	1.4938e+04	3.8320e+07	3.5018e+08
F14	Best	7.4445e+03	9.7304e+03	1.8312e+05	9.9037e+04	5.8755e+04	5.3312e+04	4.0328e+03	2.5623e+04	6.7714e+04
F14	Mean	1.4672e+05	7.6802e+05	3.5775e+06	5.6872e+05	1.4192e+06	5.3378e+05	3.8679e+04	8.1122e+05	2.3478e+06
F14	Std	2.1944e+05	5.3131e+05	3.5243e+06	3.9054e+05	9.2001e+05	5.1846e+05	4.3153e+04	8.7584e+05	1.9654e+06
F15	Best	8.1975e+03	7.9604e+03	3.2484e+05	5.0782e+04	3.8952e+05	1.5960e+04	1.9258e+03	2.1265e+04	1.7249e+05
F15	Mean	4.0386e+04	5.4354e+04	1.0880e+05	8.6205e+05	1.2904e+08	1.0233e+05	7.2418e+03	2.5564e+06	9.580e+06
F15	Std	3.7329e+04	3.7931e+04	1.6430e+07	6.1400e+05	2.0372e+08	1.1328e+05	5.6915e+03	6.4436e+06	4.5593e+06
F16	Best	1.9200e+03	2.2826e+03	3.2527e+03	2.5273e+03	2.5737e+03	2.4535e+03	2.3544e+03	2.0145e+03	2.9337e+03
F16	Mean	2.6652e+03	3.2008e+03	4.2385e+03	2.8324e+03	4.8589e+03	3.5184e+03	2.8198e+03	2.6578e+03	4.8873e+03
F16	Std	4.2206e+02	5.9352e+02	7.5334e+02	1.4745e+02	6.2938e+02	5.3960e+02	2.8781e+02	3.0790e+02	9.8390e+02
F17	Best	1.7879e+03	1.9218e+03	2.2178e+03	1.9982e+03	2.2074e+03	2.3157e+03	1.8269e+03	1.8161e+03	2.4301e+03
F17	Mean	2.1441e+03	2.4617e+03	2.6484e+03	2.1817e+03	3.0667e+03	2.6932e+03	2.3257e+03	2.1247e+03	2.9884e+03
F17	Std	2.1768e+02	4.0422e+02	2.0258e+02	1.1745e+02	8.8220e+02	2.5977e+02	2.6318e+02	2.0808e+02	4.4034e+02
F18	Best	8.3946e+04	1.8928e+05	1.1230e+05	6.1329e+05	1.1837e+06	6.1865e+05	1.1334e+05	6.5070e+04	4.4295e+05
F18	Mean	1.6822e+06	2.0020e+06	1.3313e+07	2.0412e+06	1.7012e+07	6.6485e+06	1.2579e+06	3.1621e+06	2.7479e+07
F18	Std	2.6245e+06	3.3906e+06	1.5323e+07	1.3420e+06	1.4723e+07	6.2140e+06	1.6458e+06	6.4232e+06	2.2000e+07
F19	Best	2.1282e+03	3.0706e+03	1.0316e+06	5.2496e+04	8.7927e+05	1.4686e+05	2.0146e+03	3.1402e+04	5.0136e+05
F19	Mean	1.9319e+04	1.0318e+05	2.3708e+07	6.3264e+05	4.7344e+07	2.2481e+06	1.0890e+04	5.1767e+06	1.7006e+07
F19	Std	1.8133e+04	2.2190e+05	2.3304e+07	5.6582e+05	4.1630e+07	2.0394e+06	9.0143e+03	2.1631e+07	2.5167e+07
F20	Best	2.1047e+03	2.2404e+03	2.4819e+03	2.2281e+03	2.5344e+03	2.3551e+03	2.2940e+03	2.1878e+03	2.5940e+03
F20	Mean	2.4620e+03	2.7368e+03	2.9773e+03	2.5046e+03	2.8030e+03	2.7519e+03	2.6589e+03	2.4867e+03	2.9087e+03
F20	Std	1.7492e+02	3.2993e+02	1.9815e+02	1.1777e+02	1.8632e+02	2.3249e+02	1.7339e+02	1.7191e+02	1.9951e+02
F21	Best	2.2233e+03	2.3788e+03	2.5339e+03	2.4312e+03	2.5359e+03	2.4605e+03	2.4404e+03	2.3684e+03	2.5724e+03
F21	Mean	2.3870e+03	2.4474e+03	2.6441e+03	2.4658e+03	2.6004e+03	2.5707e+03	2.4994e+03	2.4087e+03	2.689e+03
F21	Std	3.6181e+01	6.3298e+01	6.8128e+01	1.5418e+01	3.8113e+01	5.0083e+01	3.5676e+01	2.3091e+01	4.7251e+01
F22	Best	2.3055e+03	2.3367e+03	3.6830e+03	2.6723e+03	3.1673e+03	2.9313e+03	2.3133e+03	2.4584e+03	7.3355e+03
F22	Mean	2.3109e+03	2.3684e+03	8.4722e+03	3.3036e+03	7.7002e+03	7.4609e+03	5.5765e+03	4.8048e+03	9.0521e+03
F22	Std	3.8751e+00	2.0193e+01	1.5771e+03	7.7338e+02	1.3469e+03	1.6610e+03	2.0591e+03	1.9102e+03	6.7905e+02
F23	Best	2.7009e+03	2.7401e+03	2.9885e+03	2.7829e+03	3.2869e+03	2.9621e+03	2.9463e+03	2.7199e+03	3.1208e+03
F23	Mean	2.7630e+03	2.8257e+03	3.1611e+03	2.8069e+03	3.5189e+03	3.1737e+03	3.1983e+03	2.7850e+03	3.3963e+03
F23	Std	4.2157e+01	6.8307e+01	9.9183e+01	1.2607e+01	1.3686e+02	1.2870e+02	1.6368e+02	3.8608e+01	1.4147e+02
F24	Best	2.8639e+03	2.8870e+03	3.1022e+03	2.9673e+03	3.6062e+03	3.0986e+03	3.0977e+03	2.8775e+03	3.3184e+03
F24	Mean	2.9242e+03	2.9751e+03	3.2777e+03	3.0170e+03	3.8884e+03	3.2996e+03	3.2431e+03	2.9625e+03	3.6135e+03
F24	Std	3.9959e+01	5.7887e+01	1.1859e+02	1.7907e+01	1.4381e+02	1.1129e+02	8.5701e+01	6.6534e+01	1.6300e+02
F25	Best	2.8900e+03	2.9231e+03	3.0711e+03	2.8907e+03	3.5061e+03	2.9306e+03	2.8842e+03	2.9358e+03	3.1270e+03
F25	Mean	2.9273e+03	2.9556e+03	3.2289e+03	2.9122e+03	3.9315e+03	3.0702e+03	2.9140e+03	3.0233e+03	3.3767e+03
F25	Std	2.1196e+01	2.0594e+01	8.9126e+01	1.1612e+01	2.2130e+02	6.4243e+01	2.3479e+01	7.2817e+01	1.1793e+02
F26	Best	2.8185e+03	2.9975e+03	6.7042e+03	5.0276e+03	7.2857e+03	4.3059e+03	2.8396e+03	3.7370e+03	6.5569e+03
F26	Mean	4.1390e+03	4.7722e+03	8.3472e+03	5.3254e+03	9.4325e+03	8.0757e+03	5.3144e+03	4.9204e+03	9.5259e+03
F26	Std	8.1436e+02	1.1744e+03	1.1116e+03	1.3807e+02	8.8078e+02	1.5235e+03	2.2562e+03	4.1684e+02	1.0685e+03
F27	Best	3.2069e+03	3.2166e+03	3.2952e+03	3.2285e+03	3.5937e+03	3.2530e+03	3.2374e+03	3.2142e+03	3.3927e+03
F27	Mean	3.2395e+03	3.2547e+03	3.4971e+03	3.2399e+03	4.3523e+03	3.4895e+03	3.4940e+03	3.2692e+03	3.9552e+03
F27	Std	1.4931e+01	1.5344e+01	1.3487e+02	5.1738e+00	2.7311e+02	2.3945e+02	2.3793e+02	3.3349e+01	7.3709e+02
F28	Best	3.2225e+03	3.3074e+03	3.5081e+03	3.2785e+03	4.7544e+03	3.4165e+03	3.2050e+03	3.3299e+03	3.7916e+03
F28	Mean	3.3017e+03	3.3676e+03	3.9057e+03	3.3028e+03	5.				

TABLE XV
CEC2017 RESULTS AT 50 DIMENSIONS

Function	Index	IWO	WO	WOA	DE	HOA	SGA	PSO	GWO	HHO
F1	Best	4.0696e+07	6.8465e+08	1.0526e+10	3.2037e+07	6.7510e+10	1.0510e+10	1.8201e+07	3.9762e+09	3.1192e+10
F1	Mean	1.5361e+08	2.3189e+09	2.1409e+10	7.4822e+07	8.6088e+10	2.3206e+10	4.3304e+08	1.2119e+10	4.2944e+10
F1	Std	7.6136e+07	8.4378e+08	4.9099e+09	2.6052e+07	8.9733e+09	6.9825e+09	8.2191e+08	4.3263e+09	7.2379e+09
F3	Best	9.8218e+04	1.2678e+05	1.8059e+05	2.0293e+05	1.3288e+05	1.1690e+05	1.4834e+05	1.2003e+05	1.7329e+05
F3	Mean	1.7678e+05	1.9709e+05	3.0792e+05	2.7624e+05	1.5798e+05	1.9323e+05	1.7056e+05	1.6664e+05	2.2235e+05
F3	Std	5.4734e+04	3.2351e+04	9.4569e+04	3.6553e+04	1.2923e+04	3.8742e+04	2.7536e+04	2.8500e+04	2.7851e+04
F4	Best	5.5202e+02	6.8308e+02	2.8146e+03	7.0838e+02	1.2287e+04	1.7454e+03	5.0844e+02	8.7180e+02	5.5095e+03
F4	Mean	6.9971e+02	9.5107e+02	5.0475e+03	7.9260e+02	2.2321e+04	3.1825e+03	6.6748e+02	1.8421e+03	1.0136e+04
F4	Std	7.0372e+01	1.6186e+02	1.2819e+03	4.3025e+01	4.6139e+03	9.1930e+02	1.2019e+02	7.7252e+02	2.6506e+03
F5	Best	6.6250e+02	7.6789e+02	1.0015e+03	8.2915e+02	9.9150e+02	8.9480e+02	7.4775e+02	6.7831e+02	9.9532e+02
F5	Mean	7.6498e+02	8.8875e+02	1.1210e+03	8.7476e+02	1.0701e+03	9.9562e+02	8.2252e+02	7.5928e+02	1.0464e+03
F5	Std	7.5547e+01	8.3782e+01	6.5481e+01	1.9579e+01	3.1542e+01	4.4220e+01	3.9013e+01	3.8441e+01	3.7226e+01
F6	Best	6.1742e+02	6.6653e+02	6.8104e+02	6.0176e+02	6.5944e+02	6.6636e+02	6.4407e+02	6.1540e+02	6.8112e+02
F6	Mean	6.4173e+02	6.8409e+02	6.9868e+02	6.0228e+02	6.8141e+02	6.8150e+02	6.5767e+02	6.2697e+02	6.9297e+02
F6	Std	1.9260e+01	1.2728e+01	1.3557e+01	3.1600e-01	7.1873e+00	8.3994e+00	4.5850e+00	7.0933e+00	4.7721e+00
F7	Best	1.0262e+03	1.1471e+03	1.7588e+03	1.1375e+03	1.6687e+03	1.5172e+03	1.0819e+03	1.0484e+03	1.7877e+03
F7	Mean	1.1666e+03	1.3919e+03	1.9262e+03	1.1752e+03	1.8193e+03	1.8335e+03	1.2300e+03	1.1423e+03	1.9685e+03
F7	Std	1.3161e+02	1.0491e+02	8.7216e+01	2.1994e+01	8.8123e+01	1.0426e+02	7.3508e+01	6.5895e+01	6.6023e+01
F8	Best	9.4188e+02	1.0430e+03	1.2944e+03	1.1442e+03	1.3395e+03	1.2532e+03	1.0562e+03	9.9789e+02	1.3084e+03
F8	Mean	1.0533e+03	1.1956e+03	1.4111e+03	1.1850e+03	1.4049e+03	1.3207e+03	1.1451e+03	1.0582e+03	1.3635e+03
F8	Std	6.1092e+01	9.7814e+01	7.8036e+01	1.7592e+01	3.6291e+01	4.8256e+01	3.9152e+01	3.2583e+01	3.583e+01
F9	Best	3.466e+03	8.1507e+03	2.2655e+04	6.0808e+03	1.9151e+04	1.7006e+04	1.6378e+04	4.2097e+03	2.7550e+04
F9	Mean	1.9085e+04	3.1365e+04	3.7612e+04	1.0004e+04	2.7759e+04	2.4051e+04	2.5862e+04	1.2439e+04	3.4308e+04
F9	Std	1.0392e+04	1.1535e+04	9.8963e+03	2.4610e+03	4.1458e+03	3.1000e+03	5.0267e+03	5.7043e+03	3.7805e+03
F10	Best	6.4495e+03	8.4395e+03	1.1464e+04	1.1528e+04	1.1625e+04	1.0181e+04	6.1131e+03	5.6310e+03	1.1788e+04
F10	Mean	1.0825e+04	1.4946e+04	1.3376e+04	1.2712e+04	1.3012e+04	1.1011e+04	8.1232e+03	7.8695e+03	1.3718e+04
F10	Std	6.4495e+03	2.0723e+03	1.1332e+03	3.7829e+02	8.5132e+02	1.1505e+03	9.2514e+02	9.5331e+02	1.0116e+03
F11	Best	1.4152e+03	2.1915e+03	5.5154e+03	2.4119e+03	1.1312e+03	3.8125e+03	1.3487e+03	3.1207e+03	5.1857e+03
F11	Mean	1.7882e+03	3.5061e+03	9.0481e+03	7.7790e+03	1.8345e+03	7.2633e+03	1.4647e+03	7.1747e+03	1.1254e+04
F11	Std	1.7725e+02	6.7376e+02	2.1457e+03	3.0783e+03	3.0369e+03	6.9075e+03	1.2776e+03	2.5586e+03	2.5586e+03
F12	Best	9.8107e+06	7.9119e+07	1.9197e+09	1.9116e+08	3.1095e+10	9.3476e+08	1.3748e+07	1.4930e+08	1.0060e+10
F12	Mean	8.7369e+07	2.8102e+08	4.0622e+09	1.9169e+08	4.9610e+10	2.1349e+09	1.5829e+08	1.4930e+08	2.1908e+10
F12	Std	5.1513e+07	1.9040e+08	1.7454e+09	6.4867e+07	1.1865e+10	9.1076e+08	1.3246e+08	1.5954e+08	7.5865e+09
F13	Best	3.0925e+04	1.5173e+06	9.2111e+07	2.7186e+06	2.2370e+10	2.0757e+07	5.5252e+04	4.1619e+05	8.9961e+08
F13	Mean	2.9291e+05	1.4017e+07	5.7265e+08	8.0132e+06	2.2370e+10	2.4867e+08	8.6718e+06	2.5131e+08	7.9400e+09
F13	Std	2.4997e+05	1.2313e+07	3.3733e+08	4.7918e+06	8.7158e+09	2.3279e+08	4.4891e+07	2.5425e+08	4.4451e+09
F14	Best	1.9203e+05	3.6274e+05	6.9174e+05	8.0904e+05	6.4170e+06	1.6369e+05	2.9186e+04	1.7381e+05	2.5700e+06
F14	Mean	1.9734e+06	4.0700e+06	7.9402e+06	3.2195e+06	6.2046e+07	3.3803e+06	2.3127e+05	1.4498e+06	2.8291e+07
F14	Std	1.5673e+06	3.6274e+05	6.8476e+06	1.6394e+06	3.0288e+07	3.1216e+06	1.8271e+05	1.4000e+06	3.0649e+07
F15	Best	1.8387e+04	3.9712e+04	6.5794e+06	1.1519e+05	4.3558e+08	1.9120e+05	5.6159e+03	5.3986e+04	4.2496e+07
F15	Mean	1.0349e+05	1.9289e+05	7.5799e+07	1.9520e+06	3.3836e+09	7.6457e+07	1.9173e+04	1.4287e+07	4.1377e+08
F15	Std	6.8546e+04	1.3784e+05	9.0252e+07	1.5802e+06	2.0868e+09	2.7599e+08	7.0245e+03	2.3526e+07	4.0649e+08
F16	Best	2.4282e+03	2.7453e+03	4.9791e+03	3.7692e+03	5.4404e+03	3.8796e+03	2.6852e+03	2.6902e+03	4.5397e+03
F16	Mean	3.6403e+03	4.0162e+03	6.7206e+03	4.2650e+03	7.2191e+03	5.3320e+03	3.5409e+03	3.5008e+03	7.4390e+03
F16	Std	8.2946e+02	8.9072e+02	1.0291e+03	2.5305e+02	1.0011e+03	6.8920e+02	5.0493e+02	4.2032e+02	1.5913e+03
F17	Best	2.4872e+03	2.8550e+03	3.6175e+03	3.0610e+03	3.7775e+03	3.2160e+03	2.7630e+03	2.4912e+03	3.5876e+03
F17	Mean	3.3517e+03	3.9039e+03	4.6121e+03	3.4898e+03	4.7253e+03	4.2969e+03	3.2413e+03	3.0661e+03	4.6123e+03
F17	Std	4.4083e+02	6.2815e+02	6.7970e+02	1.7815e+02	5.2624e+02	5.8244e+02	3.0342e+02	3.9210e+02	9.0455e+02
F18	Best	2.4357e+05	1.1590e+06	3.8212e+06	3.8919e+06	8.9160e+06	9.5773e+05	4.6405e+05	1.2741e+06	1.1009e+07
F18	Mean	5.1423e+06	8.6522e+06	7.3053e+07	1.0860e+07	9.4603e+07	3.3324e+07	2.1604e+06	9.0655e+06	6.1411e+07
F18	Std	4.1257e+06	6.1465e+06	4.2520e+07	4.4811e+06	4.2508e+07	2.8398e+07	1.3900e+06	9.7980e+06	3.8658e+07
F19	Best	1.3343e+04	8.8289e+04	1.4252e+06	1.9547e+05	1.6205e+08	6.6247e+05	7.3212e+03	1.3737e+05	1.3589e+07
F19	Mean	1.9530e+05	6.4188e+05	1.8732e+07	5.7900e+05	1.0404e+09	2.6389e+07	2.9945e+04	2.2620e+07	1.2280e+08
F19	Std	1.9304e+05	5.2905e+05	1.7068e+07	3.3229e+05	6.0192e+08	4.3208e+07	2.1599e+04	5.6763e+07	1.0725e+08
F20	Best	2.5338e+03	2.9093e+03	3.3022e+03	3.1858e+03	3.2213e+03	3.1702e+03	2.5030e+03	2.3828e+03	3.3084e+03
F20	Mean	3.4938e+03	3.9506e+03	3.9668e+03	3.5263e+03	3.7804e+03	3.7703e+03	3.2946e+03	3.3006e+03	3.9493e+03
F20	Std	6.6813e+02	5.5718e+02	3.3613e+02	1.6724e+02	2.9940e+02	3.5139e+02	3.6770e+02	3.8675e+02	3.0146e+02
F21	Best	2.4514e+03	2.5063e+03	2.8913e+03	2.6333e+03	2.8450e+03	2.7783e+03	2.5966e+03	2.4580e+03	2.9071e+03
F21	Mean	2.5529e+03	2.6782e+03	3.0951e+03	2.6812e+03	2.9800e+03	2.9402e+03	2.7268e+03	2.5659e+03	3.1067e+03
F21	Std	9.4878e+01	1.2402e+02	1.0735e+02	1.7097e+01	5.0984e+01	2.9378e+01	6.9004e+01	7.5440e+01	9.7460e+01
F22	Best	2.3763e+03	2.9023e+03	1.2834e+04	6.0882e+03	1.3899e+04	1.1564e+04	8.0090e+03	8.0725e+03	1.2706e+04
F22	Mean	8.8235e+03	1.5259e+04	1.4802e+04	1.3799e+04	1.5328e+04	1.4026e+04	9.8570e+03	1.0302e+04	1.5345e+04
F22	Std	5.1472e+03	3.9989e+03	1.1777e+03	2.2892e+03	7.2866e+02	1.0458e+03	7.1924e+02	1.6087e+03	9.9727e+02
F23	Best	2.8783e+03	3.0204e+03	3.4480e+03	3.0649e+03	4.0601e+03	3.3627e+03	3.3467e+03	2.9358e+03	3.5949e+03
F23	Mean	2.9946e+03	3.1097e+03	3.8099e+03	3.1221e+03	4.5257e+03	3.7998e+03	3.8411e+03	3.0406e+03	4.2955e+03
F23	Std	4.9184e+01	7.2002e+01	1.7154e+02	2.4129e+01	2.6385e+02	2.5729e+02	2.8471e+02	5.6038e+01	2.8987e+02
F24	Best	3.0474e+03	3.1214e+03	3.5990e+03	3.2977e+03	4.7023e+03	3.5074e+03	3.4478e+03	3.0809e+03	4.1347e+03
F24	Mean	3.1276e+03	3.2519e+03	3.9256e+03	3.3488e+03	5.0334e+03	4.0327e+03	3.6460e+03	3.2615e+03	4.6224e+03
F24	Std	4.4159e+01	8.1294e+01	1.5488e+02	2.2738e+01	2.2776e+02	2.4875e+02	1.5400e+02	1.2116e+02	3.0360e+02
F25	Best	3.1132e+03	3.2787e+03	4.1938e+03	3.1692e+03	9.3031e+03	4.4494e+03	3.0502e+03	3.3511e+03	5.6973e+03
F25	Mean	3.1981e+03	3.5282e+03	5.4223e+03	3.2419e+03	1.1516e+04	5.1222e+03	3.1007e+03	3.9701e+03	7.2743e+03
F25	Std	4.2973e+01	1.4421e+02	6.3297e+02	4.8008e+01	1.1598e+03	4.8375e+02	2.2178e+01	3.8652e+02	6.6569e+02
F26	Best	3.5097e+03	4.7284e+03	9.6853e+03	7.1617e+03	1.3424e+04	9.0733e+03	2.9529e+03	5.5555e+03	1.1963e+04
F26	Mean	4.6038e+03	7.0589e+03	1.5011e+04	7.5769e+03	1.5714e+04	1.3329e+04	7.0656e+03	7.1761e+03	1.4331e+04
F26	Std	1.1421e+03	2.2494e+03	2.0430e+03	1.9989e+02	8.9419e+02	1.3706e+03	3.7416e+03	8.9745e+02	9.9382e+02
F27	Best	3.3625e+03	3.4981e+03	3.9331e+03	3.5176e+03	5.6701e+03	3.7778e+03	3.4507e+03	3.4766e+03	4.6294e+03
F27	Mean	3.5154e+03	3.7105e+03	4.6971e+03	3.6171e+03	7.0286e+03	4.2835e+03	4.5444e+03	3.7011e+03	6.2938e+03
F27	Std	9.2743e+01	1.1433e+02	5.3025e+02	5.2410e+01	6.5140e+02	3.5207e+02	6.2213e+02	1.0887e+02	9.4897e+02
F28	Best	3.3560e+03	3.5872e+03	5.0626e+03	3.4997e+03	9.0194e+03	4.3081e+03	3.2703e+03	3.8311e+03	6.7948e+03
F28	Mean	3.5797e+03	3.9747e+03	6.1069e+03	3.6283e+03	1.				

TABLE XVI
CEC2017 RESULTS AT 100 DIMENSIONS

Function	Index	IWO	WO	WOA	DE	HOA	SGA	PSO	GWO	HHO
F1	Best	2.8654e+09	2.2843e+10	9.1045e+10	1.3463e+09	1.9831e+11	8.5254e+10	2.4842e+08	2.6453e+10	1.3152e+11
F1	Mean	6.5239e+09	3.3442e+10	1.1195e+11	2.1820e+09	2.3111e+11	1.1141e+11	2.5268e+09	5.5806e+10	1.5284e+11
F1	Std	2.1471e+09	7.1992e+09	1.2790e+10	5.0890e+08	1.5531e+10	1.6820e+10	2.1159e+09	1.0695e+10	1.0405e+10
F3	Best	2.9503e+05	3.2694e+05	7.5051e+05	5.2981e+05	2.9890e+05	3.1362e+05	3.362e+05	3.6098e+05	3.2493e+05
F3	Mean	3.6820e+05	3.5444e+05	9.6282e+05	6.5300e+05	3.2631e+05	3.6848e+05	4.7669e+05	5.7479e+05	4.0615e+05
F3	Std	6.3582e+04	1.9171e+04	1.4832e+05	6.4677e+04	1.3362e+04	4.0754e+04	8.0313e+04	9.6514e+04	1.4551e+05
F4	Best	1.3636e+03	2.7873e+03	1.5445e+04	1.3351e+03	5.4297e+04	1.2879e+04	7.9254e+02	3.0022e+03	2.8339e+04
F4	Mean	1.7866e+03	4.3010e+03	2.2519e+04	1.7918e+03	7.0472e+04	2.0080e+04	1.0658e+03	5.6870e+03	3.8323e+04
F4	Std	2.8961e+02	1.1657e+03	3.3067e+03	1.7243e+02	8.3982e+03	4.2147e+03	3.5871e+02	1.4312e+03	6.6847e+03
F5	Best	1.0907e+03	1.3635e+03	1.8623e+03	1.4629e+03	1.8034e+03	1.7158e+03	1.2953e+03	1.1615e+03	1.7636e+03
F5	Mean	1.3146e+03	1.5874e+03	1.9855e+03	1.5738e+03	1.9283e+03	1.8228e+03	1.4041e+03	1.2680e+03	1.8875e+03
F5	Std	1.0346e+02	1.3864e+02	9.7648e+01	4.3817e+01	5.9497e+01	6.7350e+01	6.4094e+01	1.1862e+02	7.0971e+01
F6	Best	6.4497e+02	6.7082e+02	6.9350e+02	6.1213e+02	6.8977e+02	6.8135e+02	6.6408e+02	6.3680e+02	6.9628e+02
F6	Mean	6.6312e+02	7.0073e+02	7.0498e+02	6.1376e+02	6.9796e+02	6.9256e+02	6.7172e+02	6.4632e+02	7.0367e+02
F6	Std	1.1585e+01	1.5606e+01	6.8209e+00	8.5644e-01	4.2847e+00	4.8406e+00	3.4419e+00	4.9954e+00	3.8137e+00
F7	Best	1.9673e+03	2.4070e+03	3.6223e+03	1.9956e+03	3.3906e+03	3.4204e+03	2.0268e+03	1.8435e+03	3.6590e+03
F7	Mean	2.3954e+03	2.8579e+03	3.8501e+03	2.0683e+03	3.6855e+03	3.6750e+03	2.3365e+03	2.2205e+03	3.8894e+03
F7	Std	2.5321e+02	2.0013e+02	1.4662e+02	3.5048e+01	1.1637e+02	1.3412e+02	2.3548e+02	1.6184e+02	9.9316e+01
F8	Best	1.4566e+03	1.7148e+03	2.2661e+03	1.7771e+03	2.2865e+03	2.0987e+03	1.5941e+03	1.4629e+03	2.2184e+03
F8	Mean	1.6436e+03	1.9120e+03	2.4444e+03	1.8577e+03	2.3945e+03	2.2706e+03	1.7839e+03	2.3598e+03	2.5598e+03
F8	Std	1.0885e+02	1.7248e+02	1.1117e+02	3.1767e+01	7.2673e+01	9.3901e+01	9.2166e+01	5.8733e+01	7.2380e+01
F9	Best	3.3585e+04	3.9727e+04	5.5750e+04	4.8630e+04	5.3576e+04	4.3387e+04	5.7826e+04	2.3694e+04	6.1241e+04
F9	Mean	6.7916e+04	7.4039e+04	7.7977e+04	6.4175e+04	6.7959e+04	5.8295e+04	7.0093e+04	4.7125e+04	7.3239e+04
F9	Std	1.9128e+04	1.7250e+04	2.1098e+04	8.2574e+03	7.0779e+03	6.1964e+03	9.4058e+03	1.5765e+04	5.5505e+03
F10	Best	1.5097e+04	2.3256e+04	2.5125e+04	2.7987e+04	2.7056e+04	2.2324e+04	1.5426e+04	1.5365e+04	2.3575e+04
F10	Mean	2.3457e+04	3.1124e+04	2.9490e+04	2.9951e+04	2.8864e+04	2.6705e+04	1.8094e+04	1.9976e+04	2.1611e+04
F10	Std	6.9248e+03	3.5694e+03	1.6320e+03	8.0740e+02	1.2177e+03	1.0869e+03	1.3032e+03	5.2629e+03	1.7621e+03
F11	Best	2.4608e+04	7.7799e+04	1.5471e+05	7.2648e+04	1.2056e+05	1.1155e+05	2.4696e+04	5.8199e+04	1.7033e+05
F11	Mean	7.0770e+04	1.2714e+05	3.0124e+05	1.1065e+05	1.8348e+05	1.4781e+05	4.2232e+04	9.4945e+04	3.0979e+05
F11	Std	2.5190e+04	4.0596e+04	1.2176e+05	1.0585e+05	1.8348e+05	2.4903e+05	1.1743e+05	2.0348e+04	9.0861e+04
F12	Best	3.9273e+08	1.9415e+09	1.3831e+10	1.0019e+09	1.1928e+11	1.5829e+10	1.7568e+08	3.6250e+09	4.5881e+10
F12	Mean	1.0425e+09	4.0906e+09	2.9634e+10	1.7974e+09	1.4718e+11	7.3021e+09	1.7478e+09	1.2092e+10	8.1970e+10
F12	Std	3.7668e+08	1.5999e+09	8.3143e+09	3.5101e+08	1.6278e+10	1.7683e+10	1.5827e+09	6.1251e+09	1.4166e+10
F13	Best	1.4953e+05	1.9710e+07	8.9397e+08	3.6661e+05	1.6736e+10	4.5171e+08	7.1645e+05	1.6869e+08	7.9897e+09
F13	Mean	5.2728e+05	2.9600e+07	2.7899e+08	5.0467e+06	3.2296e+10	1.9015e+09	2.0737e+08	1.5709e+09	1.3277e+10
F13	Std	2.2192e+05	2.9600e+07	8.9739e+08	6.8511e+06	6.7091e+09	1.4010e+09	4.2114e+08	1.1377e+09	3.1184e+09
F14	Best	2.5893e+06	3.4446e+06	7.2138e+06	1.2883e+07	1.8971e+07	6.1322e+06	1.3370e+06	2.6271e+06	1.4155e+07
F14	Mean	7.0575e+06	1.1738e+07	2.0336e+07	2.6189e+07	4.2858e+07	1.9279e+07	3.3546e+06	9.3402e+06	2.5828e+07
F14	Std	3.2189e+06	4.4252e+06	7.7842e+06	9.5431e+06	2.3131e+07	1.0439e+07	1.0353e+06	6.1897e+06	7.6800e+06
F15	Best	6.3429e+04	6.3104e+05	8.4838e+07	7.4490e+05	9.1141e+09	3.0771e+07	7.7891e+04	4.3288e+06	9.3614e+08
F15	Mean	1.6142e+05	9.2958e+06	4.8856e+08	4.3216e+06	1.4833e+10	3.5558e+08	2.0959e+05	3.6610e+08	4.3126e+09
F15	Std	1.4908e+05	1.5739e+07	2.4723e+08	3.6888e+06	2.9574e+09	7.7406e+08	1.5133e+05	4.7535e+08	1.8089e+09
F16	Best	4.8249e+03	7.0119e+03	1.2304e+04	9.7748e+03	1.2574e+04	1.0944e+04	4.8831e+03	5.3717e+03	1.3238e+04
F16	Mean	6.9247e+03	8.2271e+03	1.7192e+04	1.0479e+04	1.7605e+04	1.4064e+04	6.5809e+03	6.9107e+03	1.7500e+04
F16	Std	1.5923e+03	9.4545e+02	2.7502e+03	3.3105e+02	2.2959e+03	1.7026e+03	6.4205e+02	1.1001e+03	2.2794e+03
F17	Best	4.2191e+03	5.5662e+03	9.5497e+03	6.5501e+03	7.8799e+04	7.2297e+03	4.4356e+03	4.5941e+03	1.3891e+04
F17	Mean	6.2092e+03	6.5574e+03	4.0049e+04	7.2576e+03	2.1294e+06	1.3870e+04	5.3839e+03	5.8440e+03	3.2489e+05
F17	Std	1.0468e+03	8.2074e+02	4.7602e+04	3.0139e+02	1.8716e+06	7.1133e+03	5.3111e+02	1.7888e+03	4.1504e+05
F18	Best	2.5710e+06	5.1984e+06	2.8322e+06	1.8026e+07	1.7426e+07	6.3041e+06	1.1539e+06	1.4083e+06	6.9539e+06
F18	Mean	7.7539e+06	1.5841e+07	1.9893e+07	3.8036e+07	8.3497e+07	1.6657e+07	4.4946e+06	1.2946e+07	3.865e+07
F18	Std	3.0671e+06	9.0284e+06	8.3863e+06	9.9578e+06	4.8287e+07	8.0900e+06	3.0057e+06	1.0633e+07	2.1242e+07
F19	Best	1.0798e+05	3.2370e+06	1.2519e+08	9.1427e+05	6.6817e+09	3.5463e+07	2.2520e+05	2.2074e+07	1.6932e+09
F19	Mean	2.2735e+06	1.5286e+07	5.3327e+08	4.6746e+06	1.5365e+10	2.2462e+08	3.2075e+06	3.3485e+08	5.0819e+09
F19	Std	1.4557e+06	7.3376e+06	3.1117e+08	3.1683e+06	4.8305e+09	3.5903e+08	1.9354e+06	3.9246e+08	2.1236e+09
F20	Best	3.8880e+03	5.0430e+03	5.9276e+03	6.2414e+03	5.5854e+03	4.9021e+03	4.4065e+03	4.2783e+03	6.3410e+03
F20	Mean	6.5900e+03	7.2679e+03	7.1472e+03	6.8640e+03	6.8963e+03	6.4606e+03	5.4236e+03	5.7650e+03	7.1292e+03
F20	Std	1.4873e+03	1.0946e+03	5.9888e+02	2.6693e+02	5.8078e+02	7.5635e+02	5.5178e+02	1.0983e+03	4.0308e+02
F21	Best	2.9711e+03	3.0140e+03	4.1014e+03	3.3138e+03	4.1007e+03	3.8927e+03	3.2641e+03	2.9782e+03	4.3714e+03
F21	Mean	3.1171e+03	3.4253e+03	4.4475e+03	3.3788e+03	4.4292e+03	4.4089e+03	3.7378e+03	3.1066e+03	4.7435e+03
F21	Std	1.1127e+02	1.5537e+02	1.8459e+02	3.2401e+01	1.4768e+02	2.3869e+02	2.1256e+02	7.8083e+01	2.5316e+02
F22	Best	3.1765e+03	5.9452e+03	2.8119e+04	3.1097e+04	2.9549e+04	2.6801e+04	1.9292e+04	1.7743e+04	2.9636e+04
F22	Mean	2.3791e+04	2.9665e+04	3.1935e+04	3.2514e+04	3.1860e+04	2.9746e+04	2.1951e+04	2.2426e+04	3.2176e+04
F22	Std	8.9104e+03	7.7897e+03	1.8330e+03	6.0650e+02	1.0247e+03	1.7028e+03	1.2668e+03	4.3056e+03	1.3531e+03
F23	Best	3.4036e+03	3.6827e+03	4.8803e+03	3.6615e+03	6.6716e+03	4.9982e+03	4.6252e+03	3.5248e+03	5.3681e+03
F23	Mean	3.5726e+03	3.8975e+03	5.3087e+03	3.7214e+03	7.4545e+03	5.6326e+03	5.3647e+03	3.7764e+03	6.2691e+03
F23	Std	1.4129e+02	1.6980e+02	2.1523e+02	2.9805e+01	3.6169e+02	3.8663e+02	4.0753e+02	1.5518e+02	5.1170e+02
F24	Best	3.9268e+03	4.2536e+03	6.1441e+03	4.2393e+03	9.7338e+03	6.1126e+03	4.6469e+03	4.2013e+03	8.3394e+03
F24	Mean	4.1635e+03	4.6125e+03	6.8037e+03	4.3128e+03	1.1857e+04	7.1160e+03	5.5794e+03	4.5002e+03	9.6643e+03
F24	Std	1.5935e+02	1.6347e+02	3.9799e+02	4.2393e+03	9.3591e+02	4.6596e+02	3.8919e+02	1.4904e+02	7.7807e+02
F25	Best	4.1880e+03	5.5054e+03	9.0252e+03	4.6093e+03	1.9894e+04	9.0780e+03	3.4022e+03	4.9209e+03	1.2215e+04
F25	Mean	4.5190e+03	6.4791e+03	1.0973e+04	4.9036e+03	2.4059e+04	1.1108e+04	3.5477e+03	7.0387e+03	1.4554e+04
F25	Std	1.7042e+02	6.9227e+02	1.2108e+03	2.4541e+02	2.2998e+03	1.1810e+03	5.9957e+01	9.6639e+02	1.1275e+03
F26	Best	8.0007e+03	1.8390e+04	3.2467e+04	1.5934e+04	4.3034e+04	3.0364e+04	3.9861e+03	1.4751e+04	3.5013e+04
F26	Mean	1.5958e+04	2.1438e+04	3.9433e+04	1.6470e+04	4.8539e+04	3.6764e+04	1.8804e+04	7.1793e+04	4.0787e+04
F26	Std	3.1740e+03	2.7873e+03	3.2287e+03	1.5934e+04	3.0508e+03	3.9547e+03	8.8734e+03	1.2139e+03	2.7492e+03
F27	Best	3.5963e+03	3.8670e+03	4.9544e+03	4.1186e+03	1.0802e+04	4.4801e+03	3.7774e+03	3.9419e+03	6.6449e+03
F27	Mean	3.8184e+03	4.1513e+03	6.7008e+03	4.2847e+03	1.3218e+04	5.7793e+03	4.1674e+03	4.2662e+03	1.1384e+04
F27	Std	1.0465e+02	2.1464e+02	1.0138e+03	1.0193e+02	9.2838e+02	6.6449e+02	2.3125e+02	2.3032e+02	2.3557e+03
F28	Best	4.1558e+03	6.2628e+03	1.2369e+04	5.2463e+03	2.7429e+04	1.3152e+04	3.5490e+03	7.8282e+03	1.4810e+04
F28	Mean	5.2245e+03	7.9592e+03	1.4307e+04	6.1651e+03					

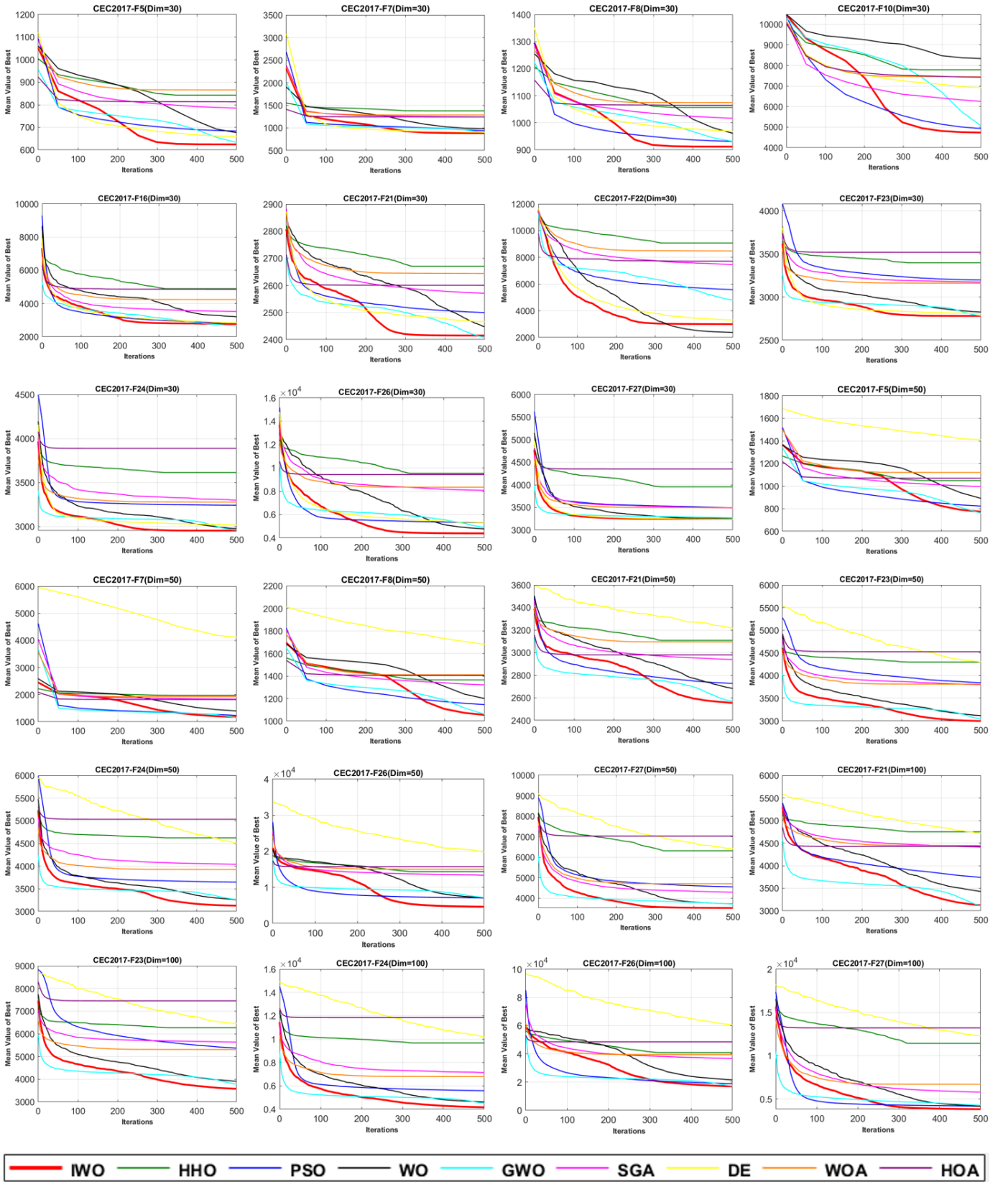


Fig. 6. Convergence curves of different optimizers on representative CEC2017 benchmark functions

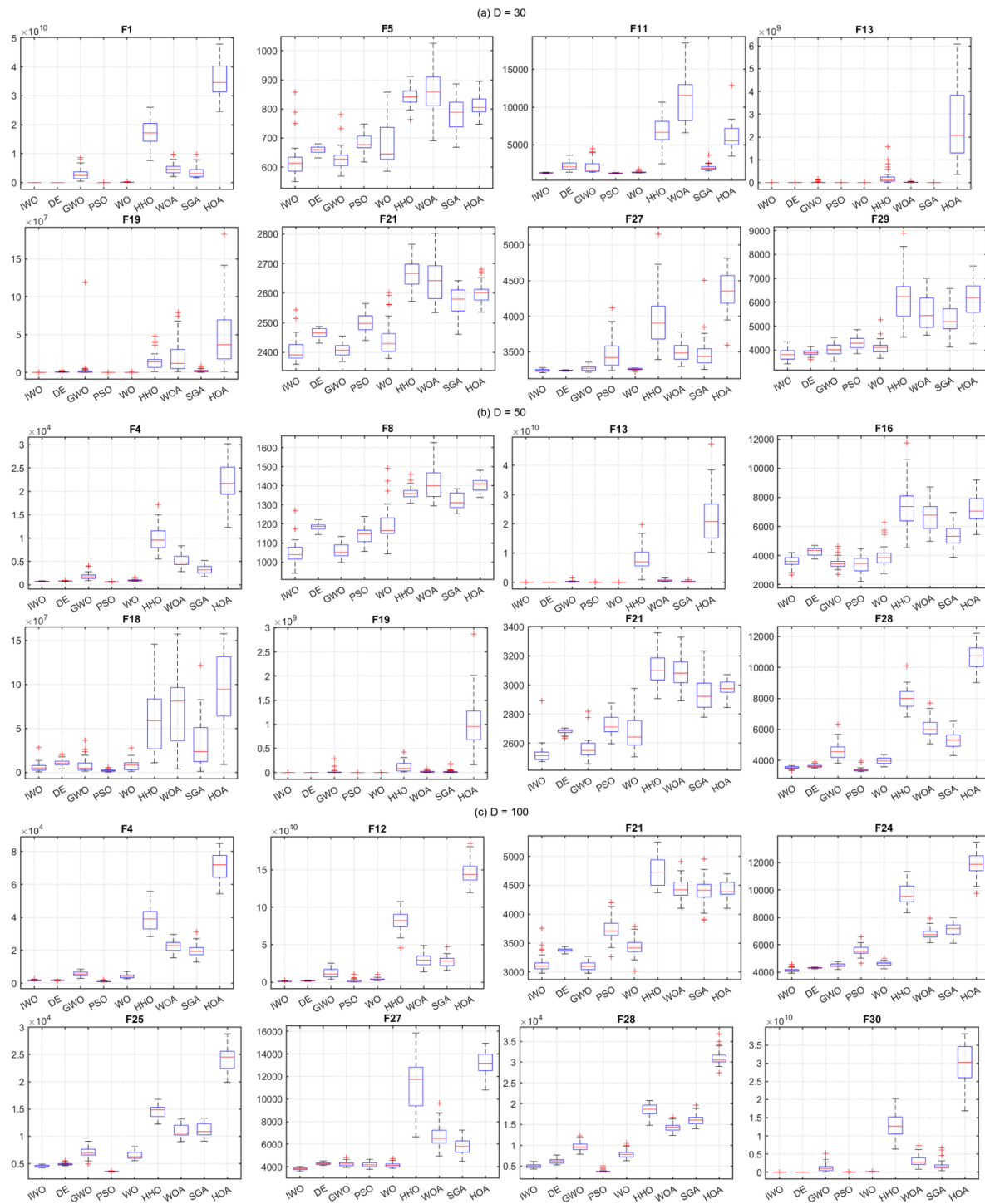


Fig. 7. Boxplots of optimization results on representative CEC2017 benchmark functions at different dimensions: (a) D=30, (b) D=50, and (c) D=100

TABLE XVII
FUNCTION-WISE WILCOXON RANK-SUM TEST OUTCOMES FOR CEC2017 ACROSS DIFFERENT DIMENSIONS

Function	WO	WOA	DE	HOA	SGA	PSO	GWO	HHO
F1	(+,+,+)	(+,+,+)	(+,+,=)	(+,+,+)	(+,+,+)	(+,+,-)	(+,+,+)	(+,+,+)
F3	(+,+,=)	(+,+,+)	(+,+,+)	(+,+,-)	(+,+,+)	(+,+,+)	(+,+,+)	(+,+,=)
F4	(+,+,+)	(+,+,+)	(+,+,+)	(+,+,+)	(+,+,+)	(=,-,-)	(+,+,+)	(+,+,+)
F5	(+,+,+)	(+,+,+)	(+,+,+)	(+,+,+)	(+,+,+)	(+,+,+)	(=,=,=)	(+,+,+)
F6	(+,+,+)	(+,+,+)	(-,-,-)	(+,+,+)	(+,+,+)	(+,+,+)	(-,-,-)	(+,+,+)
F7	(+,+,+)	(+,+,+)	(+,+,-)	(+,+,+)	(+,+,+)	(+,+,=)	(=,=,-)	(+,+,+)
F8	(+,+,+)	(+,+,+)	(+,+,+)	(+,+,+)	(+,+,+)	(+,+,+)	(=,=,-)	(+,+,+)
F9	(+,+,+)	(+,+,+)	(-,-,+)	(+,+,+)	(+,+,+)	(+,+,+)	(-,-,+)	(+,+,+)
F10	(+,+,+)	(+,+,+)	(+,+,+)	(+,+,+)	(+,+,+)	(=,=,=)	(=,=,=)	(+,+,+)
F11	(+,+,+)	(+,+,+)	(+,+,+)	(+,+,+)	(+,+,+)	(-,-,=)	(+,+,+)	(+,+,+)
F12	(+,+,+)	(+,+,+)	(+,+,+)	(+,+,+)	(+,+,+)	(-,-,+)	(+,+,+)	(+,+,+)
F13	(+,+,+)	(+,+,+)	(+,+,+)	(+,+,+)	(+,+,+)	(-,-,+)	(+,+,+)	(+,+,+)
F14	(+,+,+)	(+,+,+)	(+,+,+)	(+,+,+)	(+,+,+)	(-,-,-)	(+,+,+)	(+,+,+)
F15	(=,+,+)	(+,+,+)	(+,+,+)	(+,+,+)	(=,+,+)	(-,-,+)	(+,+,+)	(+,+,+)
F16	(+,+,+)	(+,+,+)	(=,+,+)	(+,+,+)	(+,+,+)	(=,=,=)	(=,=,=)	(+,+,+)
F17	(=,+,+)	(+,+,+)	(-,-,+)	(+,+,+)	(+,+,+)	(=,=,-)	(-,-,=)	(+,+,+)
F18	(=,+,+)	(+,+,+)	(+,+,+)	(+,+,+)	(+,+,+)	(=,=,=)	(=,=,=)	(+,+,+)
F19	(=,+,+)	(+,+,+)	(+,+,-)	(+,+,+)	(+,+,+)	(-,-,-)	(+,+,+)	(+,+,+)
F20	(+,+,+)	(+,+,+)	(-,-,+)	(+,+,+)	(+,+,+)	(=,=,=)	(-,-,=)	(+,+,+)
F21	(+,+,+)	(+,+,+)	(+,+,+)	(+,+,+)	(+,+,+)	(+,+,+)	(=,=,-)	(+,+,+)
F22	(+,+,+)	(+,+,+)	(+,+,+)	(+,+,+)	(+,+,+)	(+,+,+)	(+,+,+)	(+,+,+)
F23	(=,=,=)	(+,+,+)	(+,+,+)	(+,+,+)	(+,+,+)	(+,+,+)	(=,=,=)	(+,+,+)
F24	(+,+,+)	(+,+,+)	(+,+,+)	(+,+,+)	(+,+,+)	(+,+,+)	(=,=,=)	(+,+,+)
F25	(+,+,+)	(+,+,+)	(=,+,+)	(+,+,+)	(+,+,+)	(=,=,-)	(+,+,+)	(+,+,+)
F26	(=,+,+)	(+,+,+)	(+,+,-)	(+,+,+)	(+,+,+)	(+,+,+)	(=,=,=)	(+,+,+)
F27	(=,=,=)	(+,+,+)	(=,=,=)	(+,+,+)	(+,+,+)	(+,+,+)	(=,=,=)	(+,+,+)
F28	(+,+,+)	(+,+,+)	(+,+,+)	(+,+,+)	(+,+,+)	(-,-,-)	(+,+,+)	(+,+,+)
F29	(=,+,+)	(+,+,+)	(=,=,=)	(+,+,+)	(+,+,+)	(+,+,+)	(=,=,=)	(+,+,+)
F30	(+,+,+)	(+,+,+)	(=,=,-)	(+,+,+)	(+,+,+)	(-,-,-)	(+,+,+)	(+,+,+)
+/-/ (D=30)	21/0/8	29/0/0	20/4/5	29/0/0	28/0/1	14/8/7	13/4/12	29/0/0
+/-/ (D=50)	26/0/3	29/0/0	23/2/4	29/0/0	29/0/0	13/10/6	15/4/10	29/0/0
+/-/ (D=100)	28/0/1	29/0/0	23/5/1	28/1/0	29/0/0	14/8/7	17/4/8	28/0/1

Note: Entries are ordered as (D=30,D=50,D=100).

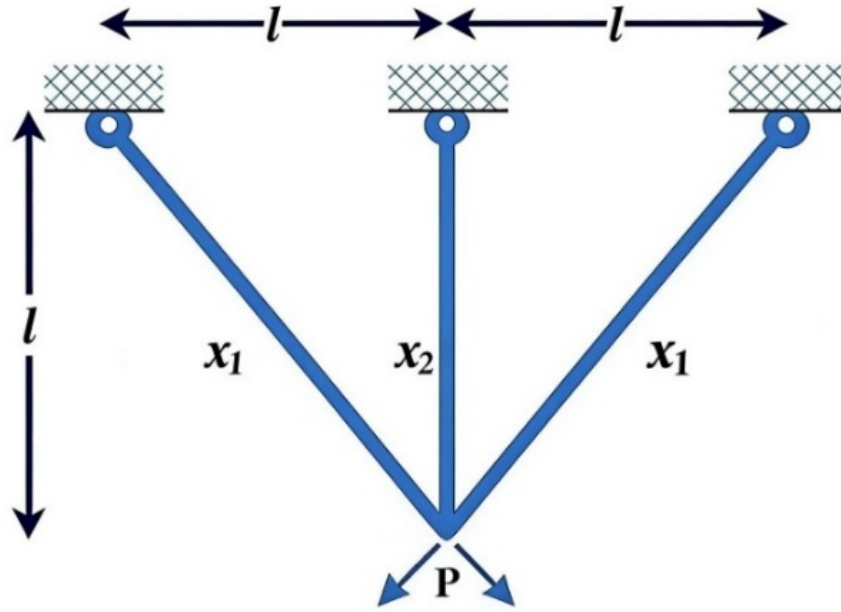


Fig. 8. Schematic diagram of the three-bar truss design problem

APPENDIX C

The mathematical model of the three-bar truss design problem is formulated as follows:

Minimize

$$f(x) = l \cdot (2\sqrt{2}x_1 + x_2) \quad (C1)$$

Subject to

$$\begin{cases} g_1(x) = \frac{2(\sqrt{2}x_1 + x_2)}{\sqrt{2}x_1^2 + 2x_1x_2} - 2 \leq 0 \\ g_2(x) = \frac{2x_2}{\sqrt{2}x_1^2 + 2x_1x_2} - 2 \leq 0 \\ g_3(x) = \frac{2}{\sqrt{2}x_2 + x_1} - 2 \leq 0 \end{cases} \quad (C2)$$

where $l = 100$ denotes the length of each member, and x_1 and x_2 are continuous design variables within the interval $[0, 1]$. The constraints $g_1(x)$, $g_2(x)$, and $g_3(x)$ represent the normalized constraints of the three-bar truss structure.

Table XVIII reports the comparative results, with the ranking determined by the mean objective value.

Tension/compression spring design problem

The tension/compression spring design problem minimizes spring weight subject to shear-stress, surge-frequency, and deflection constraints. The structural configuration is shown in Fig. 9.

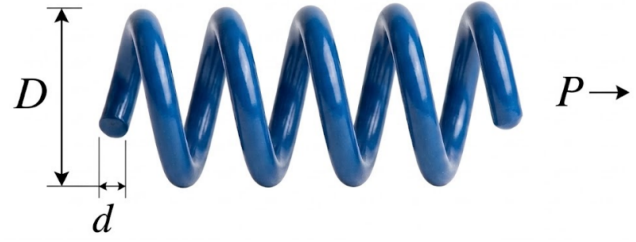


Fig. 9. Schematic diagram of the tension/compression spring design problem

The mathematical model of the tension/compression spring design problem is formulated as follows:

Minimize

$$f(x) = (x_3 + 2)x_1^2x_2 \quad (C3)$$

Subject

to

$$\begin{cases} g_1(x) = 1 - \frac{x_2^3x_3}{71785x_1^4} \leq 0 \\ g_2(x) = \frac{4x_2^2 - x_1x_2}{12566(x_2x_1^3 - x_1^4)} + \frac{1}{5108x_1^2} - 1 \leq 0 \\ g_3(x) = 1 - \frac{140.45x_1}{x_2^2x_3} \leq 0 \\ g_4(x) = \frac{x_2 + x_1}{1.5} - 1 \leq 0 \end{cases} \quad (C4)$$

where x_1 , x_2 , and x_3 denote the wire diameter, mean coil diameter, and number of active coils, respectively. The design-variable bounds are retained according to the standard benchmark definition.

Table XIX reports the comparative results, with the ranking determined by the mean objective value.

Speed reducer design problem

The speed reducer design problem minimizes the reducer weight by optimizing seven design variables subject to geometry, stress, and shaft-deflection constraints. The structural configuration is shown in Fig. 10.

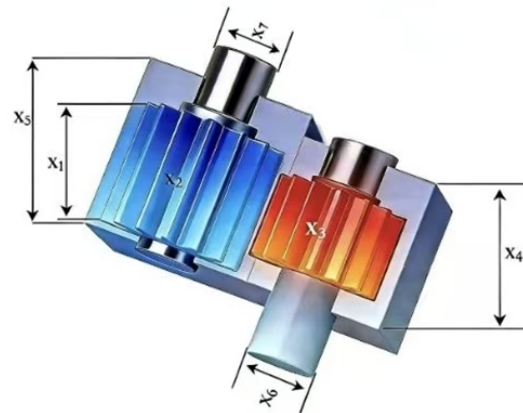


Fig. 10. Schematic diagram of the speed reducer design problem

The speed reducer contains seven design variables. x_1 , x_2 , and x_3 denote the face width, tooth module, and number of pinion teeth, respectively. x_4 and x_5 denote the bearing spans of shafts 1 and 2, while x_6 and x_7 denote the corresponding shaft diameters.

TABLE XVIII
COMPARISON OF OPTIMIZATION RESULTS FOR THE THREE-BAR TRUSS DESIGN PROBLEM

Algorithm	Mean	Std	Optimal	x_1	x_2	Rank(Mean)
IWO	2.6390e+02	7.4018e-03	2.6390e+02	0.7886	0.4085	1
WO	2.6393e+02	5.3434e-02	2.6390e+02	0.7890	0.4073	3
PSO	2.6453e+02	3.4589e+00	2.6390e+02	0.7887	0.4082	7
WOA	2.6599e+02	3.7129e+00	2.6390e+02	0.7900	0.4046	8
DE	2.6394e+02	3.8470e-02	2.6390e+02	0.7866	0.4140	4
SGA	2.6862e+02	6.1415e+00	2.6390e+02	0.7876	0.4112	9
GWO	2.6392e+02	6.6106e-03	2.6390e+02	0.7889	0.4076	2
HHO	2.6438e+02	7.3754e-01	2.6390e+02	0.7879	0.4104	6
HOA	2.6400e+02	1.6455e-01	2.6390e+02	0.7877	0.4110	5

TABLE XIX
COMPARISON OF OPTIMIZATION RESULTS FOR THE TENSION/COMPRESSION SPRING DESIGN PROBLEM

Algorithm	Mean	Std	Optimal	x_1	x_2	x_3	Rank
IWO	1.3691e-02	1.3325e-03	1.2665e-02	0.0518	0.3595	11.1298	2
WO	1.4062e-02	1.8461e-03	1.2685e-02	0.0527	0.3824	9.9263	6
PSO	1.4366e-02	4.4727e-03	1.2695e-02	0.0530	0.3881	9.6596	9
WOA	1.3950e-02	1.4543e-03	1.2671e-02	0.0511	0.3436	12.0987	4
DE	1.4111e-02	1.0116e-03	1.2789e-02	0.0532	0.3919	9.5494	7
SGA	1.4021e-02	1.5107e-03	1.2720e-02	0.0500	0.3174	14.0285	5
GWO	1.2822e-02	1.5576e-04	1.2691e-02	0.0521	0.3673	10.7105	1
HHO	1.3720e-02	1.0458e-03	1.2742e-02	0.0538	0.4089	8.7762	3
HOA	1.4129e-02	6.8449e-04	1.3001e-02	0.0556	0.4552	7.2558	8

The design vector is defined as $x = [x_1, x_2, x_3, x_4, x_5, x_6, x_7]$, where x_1 is the face width, x_2 is the module of teeth, x_3 is the number of teeth on the pinion, x_4 and x_5 are the lengths between bearings for shaft 1 and shaft 2, respectively, and x_6 and x_7 are the diameters of shaft 1 and shaft 2, respectively. The mathematical model of the speed reducer design problem is formulated as follows:

Minimize

$$f(x) = 0.7854x_1x_2^2(3.3333x_3^2 + 14.9334x_3 - 43.0934) - 1.508x_1(x_6^2 + x_7^2) + 7.477(x_6^3 + x_7^3) + 0.7854(x_4x_6^2 + x_5x_7^2) \quad (C5)$$

Subject to

$$\left\{ \begin{array}{l} g_1(x) = \frac{27}{x_1x_2^2x_3} - 1 \leq 0 \\ g_2(x) = \frac{397.5}{x_1x_2^2x_3^2} - 1 \leq 0 \\ g_3(x) = \frac{1.93x_3^3}{x_2x_3x_6^4} - 1 \leq 0 \\ g_4(x) = \frac{1.93x_3^3}{x_2x_3x_7^4} - 1 \leq 0 \\ g_5(x) = \frac{\sqrt{16.91 \cdot 10^6 + (\frac{745x_4}{x_2x_3})^2}}{110x_6^3} - 1 \leq 0 \\ g_6(x) = \frac{\sqrt{157.5 \cdot 10^6 + (\frac{745x_5}{x_2x_3})^2}}{85x_7^3} - 1 \leq 0 \\ g_7(x) = \frac{x_2x_3}{40} - 1 \leq 0 \\ g_8(x) = \frac{5x_2}{x_1} - 1 \leq 0 \\ g_9(x) = \frac{x_1}{12x_2} - 1 \leq 0 \\ g_{10}(x) = \frac{1.5x_6 + 1.9}{x_4} - 1 \leq 0 \\ g_{11}(x) = \frac{1.1x_7 + 1.9}{x_5} - 1 \leq 0 \end{array} \right. \quad (C6)$$

where the bounds of the design variables are $x_1 \in [2.6, 3.6]$, $x_2 \in [0.7, 0.8]$, $x_3 \in [17, 18]$, $x_4, x_5 \in [7.3, 8.3]$, $x_6 \in [2.9, 3.9]$, and $x_7 \in [5.0, 5.5]$.

Table XX reports the comparative results, with the ranking determined by the mean objective value.

Car side impact design problem

The car side-impact design problem minimizes vehicle weight subject to structural-response and occupant-safety constraints. This problem involves 11 design variables and 10 nonlinear constraints related to crashworthiness performance, including dummy abdominal load, rib compression deformation, viscous criterion, and structural intrusion velocity. The design configuration is shown in Fig. 11.

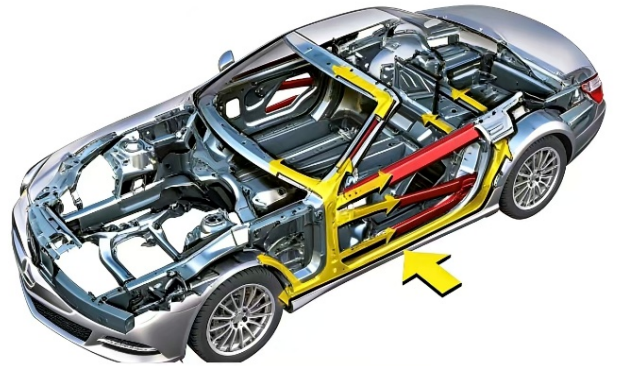


Fig. 11. Schematic diagram of the car side impact design problem

The optimization model is formulated as follows:
Minimize

$$f(x) = 1.98 + 4.90x_1 + 6.67x_2 + 6.98x_3 + 4.01x_4 + 1.78x_5 + 2.73x_7 \quad (C7)$$

Subject to

$$\left\{ \begin{array}{l} g_1(x) = F_a - 1 \leq 0 \\ F_a = 1.16 - 0.3717x_2x_4 - 0.00931x_2x_{10} - 0.484x_3x_9 + 0.01343x_6x_{10} \\ g_2(x) = VC_c - 0.32 \leq 0 \end{array} \right.$$

TABLE XX
COMPARISON OF OPTIMIZATION RESULTS FOR THE SPEED REDUCER DESIGN PROBLEM

Algorithm	Mean	Std	Optimal	x_1	x_2	x_3	x_4	x_5	x_6	x_7	Rank
IWO	2.9944e+03	3.4538e-02	2.9944e+03	3.5000	0.7000	17.0000	7.3000	7.7153	3.3505	5.2867	1
WO	2.9980e+03	3.1981e+00	2.9946e+03	3.5000	0.7000	17.0000	7.3084	7.7170	3.3506	5.2867	3
PSO	3.2126e+03	4.6056e+02	3.0167e+03	3.5000	0.7000	17.0000	8.3000	8.3000	3.3525	5.2869	6
WOA	3.5033e+03	6.6281e+02	3.0044e+03	3.5198	0.7000	17.0000	7.3000	7.7221	3.3505	5.2899	7
DE	2.9945e+03	2.3596e-01	2.9944e+03	3.5000	0.7000	17.0000	7.3000	7.7153	3.3505	5.2867	2
SGA	3.0025e+03	5.1705e+00	2.9945e+03	3.5000	0.7000	17.0001	7.3004	7.7167	3.3506	5.2867	4
GWO	3.0110e+03	5.0847e+00	3.0020e+03	3.5029	0.7000	17.0000	7.3000	7.8901	3.3551	5.2889	5
HHO	4.0129e+03	7.8712e+02	3.0305e+03	3.5000	0.7000	17.0000	7.4061	8.0270	3.3588	5.3273	9
HOA	3.9676e+06	9.2384e+06	3.1005e+03	3.5689	0.7000	17.0000	8.2070	7.9541	3.3831	5.3752	8

where x_1 , x_3 , and x_4 are limited to the range $[0.5, 1.5]$, x_2 is restricted to $[0.45, 1.35]$, x_5 is restricted to $[0.875, 2.625]$. x_6 and x_7 are limited to $[0.4, 1.2]$. In addition, x_8 and x_9 are selected within the interval of $[0.192, 0.345]$, x_{10} is limited to $[0, 30]$, and x_{11} is restricted to the range of $[-30, 30]$.

Table XXI reports the comparative results, with the ranking determined by the mean objective value.

TABLE XXI
COMPARISON OF OPTIMIZATION RESULTS FOR THE CAR SIDE IMPACT DESIGN PROBLEM

Algorithm	Mean	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	x_9	x_{10}	x_{11}	Rank
IWO	2.2843e+01	0.5000	1.1174	0.5000	1.3005	0.5000	1.5000	0.5000	0.7598	0.9995	-19.3760	-0.1759	1
WO	2.2854e+01	0.5000	1.1315	0.5000	1.2798	0.5000	1.5000	0.5000	0.5543	0.5467	-16.8494	0.1716	4
PSO	2.2846e+01	0.5000	1.1110	0.5000	1.3117	0.5000	1.5000	0.5000	1.0000	1.0000	-20.5249	-0.4590	2
WOA	2.3891e+01	0.5000	1.2420	0.8361	0.7695	0.5000	1.5000	0.5000	0.0713	0.7172	-30.0000	-0.5075	9
DE	2.2996e+01	0.5000	1.1401	0.5201	1.2659	0.5000	1.4984	0.5001	0.7304	0.0298	-16.9828	-1.0654	6
SGA	2.2869e+01	0.5001	1.1323	0.5003	1.2804	0.5024	1.5000	0.5000	0.8696	1.0000	-16.8038	1.1311	5
GWO	2.2853e+01	0.5002	1.1243	0.5000	1.2913	0.5000	1.5000	0.5000	0.7527	0.0735	-18.1823	1.1532	3
HHO	2.3437e+01	0.5000	1.3707	0.5000	1.0273	0.5000	0.5000	0.5000	0.6798	0.0850	25.8813	25.3852	7
HOA	2.3856e+01	0.5505	1.1463	0.5189	1.3097	0.5928	1.3258	0.5877	0.5708	0.9611	-10.4099	3.0581	8

APPENDIX D

Table XXII reports the function-wise ablation results.

TABLE XXII
ABLATION RESULTS OF IWO VARIANTS ON THE 50-DIMENSIONAL CEC2017 BENCHMARK SUITE

Function	Index	WO	IWO-noS1	IWO-noS2	IWO-noS3	IWO-noS4	IWO-noS5	IWO
F1	Mean	2.3711e+09	1.4533e+08	2.6117e+07	5.3487e+07	3.3907e+07	3.2238e+07	7.9427e+06
F1	Std	1.4206e+09	6.7388e+07	1.5923e+07	3.4740e+07	2.3075e+07	2.0466e+07	6.7427e+06
F1	Rank	6	2	2	5	4	3	1
F3	Mean	2.1049e+05	1.6594e+05	1.3927e+05	1.8828e+05	1.5018e+05	1.5641e+05	1.4661e+05
F3	Std	4.0052e+04	4.0857e+04	2.6106e+04	4.4079e+04	3.1754e+04	3.3090e+04	3.1830e+04
F3	Rank	7	5	1	6	3	4	2
F4	Mean	5.7742e+02	3.2446e+02	2.9542e+02	3.1775e+02	2.9650e+02	3.1128e+02	2.7906e+02
F4	Std	1.7024e+02	7.3767e+01	6.7252e+01	8.0551e+01	6.9932e+01	7.2807e+01	6.0273e+01
F4	Rank	6	5	2	5	3	4	1
F5	Mean	4.1509e+02	2.9201e+02	2.7714e+02	2.8545e+02	2.9878e+02	2.8386e+02	2.6168e+02
F5	Std	1.2852e+02	3.7693e+01	5.8376e+01	4.6052e+01	5.1432e+01	5.3556e+01	3.5656e+01
F5	Rank	7	5	2	4	3	2	1
F6	Mean	8.2312e+01	4.5819e+01	4.5688e+01	4.6202e+01	4.5971e+01	4.4381e+01	4.2144e+01
F6	Std	1.3640e+01	8.8429e+00	9.0007e+00	9.8217e+00	1.0955e+01	1.0169e+01	1.0197e+01
F6	Rank	7	4	3	6	5	2	1
F7	Mean	7.2790e+02	5.5572e+02	4.6882e+02	5.0710e+02	4.7169e+02	4.7439e+02	4.2210e+02
F7	Std	1.1585e+02	1.3583e+02	1.0073e+02	8.5524e+01	7.3567e+01	9.8022e+01	1.2686e+02
F7	Rank	7	6	2	5	3	4	1
F8	Mean	3.7178e+02	2.8784e+02	2.8371e+02	3.0061e+02	2.8076e+02	2.7655e+02	2.7687e+02
F8	Std	9.0450e+01	4.1966e+01	4.9752e+01	5.0352e+01	4.7459e+01	5.7531e+01	5.1343e+01
F8	Rank	7	5	4	3	2	1	2
F9	Mean	2.6391e+04	1.1422e+04	1.0362e+04	1.1711e+04	1.0691e+04	1.0897e+04	1.0094e+04
F9	Std	1.1077e+04	2.5348e+03	2.4660e+03	3.1681e+03	3.2052e+03	2.9584e+03	3.1507e+03
F9	Rank	7	5	6	3	2	1	4
F10	Mean	1.3537e+04	6.9153e+03	6.8071e+03	7.0101e+03	6.4898e+03	6.9133e+03	6.6082e+03
F10	Std	2.2599e+03	1.2244e+03	8.2945e+02	9.2945e+02	6.7004e+02	1.0908e+03	1.1057e+03
F10	Rank	7	3	2	1	4	2	5
F11	Mean	2.8614e+03	7.0586e+02	5.3642e+02	5.9597e+02	5.6925e+02	5.4477e+02	4.6604e+02
F11	Std	1.2060e+03	2.4538e+02	1.1698e+02	1.2798e+02	1.3048e+02	1.2723e+02	1.0991e+02
F11	Rank	7	6	2	5	4	3	1
F12	Mean	3.0447e+08	1.0990e+08	9.0452e+07	1.2908e+08	9.1505e+07	7.9105e+07	8.9784e+07
F12	Std	1.6449e+08	5.9560e+07	5.7344e+07	7.3257e+07	6.4133e+07	5.1874e+07	3.9246e+07
F12	Rank	7	5	3	6	4	1	2
F13	Mean	1.0368e+07	3.5202e+05	2.2754e+05	2.3179e+05	2.0895e+05	2.5310e+05	2.2676e+05
F13	Std	1.2611e+07	5.3638e+05	2.0917e+05	1.9514e+05	1.3226e+05	2.1327e+05	1.3140e+05
F13	Rank	7	6	4	3	2	1	5
F14	Mean	3.4468e+06	2.0040e+06	1.2746e+06	1.4314e+06	1.6534e+06	1.2839e+06	8.8844e+05
F14	Std	2.5323e+06	1.9292e+06	1.0016e+06	9.8875e+05	1.7291e+06	1.0797e+06	5.4590e+05
F14	Rank	7	6	2	5	4	1	3
F15	Mean	3.6441e+05	6.8278e+04	8.8869e+04	1.0204e+05	7.7149e+04	7.7526e+04	7.7713e+04
F15	Std	2.8233e+05	3.5589e+04	6.4985e+04	5.3805e+04	5.0949e+04	4.5076e+04	5.9894e+04
F15	Rank	7	5	3	6	2	1	4
F16	Mean	2.7355e+03	2.0820e+03	2.0733e+03	2.1727e+03	2.1199e+03	2.1610e+03	2.0976e+03
F16	Std	1.0955e+03	5.2752e+02	4.4679e+02	3.9916e+02	4.4681e+02	4.1889e+02	4.9120e+02
F16	Rank	7	2	1	6	4	3	5
F17	Mean	1.9340e+03	1.5044e+03	1.6265e+03	1.5358e+03	1.6128e+03	1.6181e+03	1.4799e+03
F17	Std	5.8550e+02	3.6760e+02	3.0501e+02	4.1765e+02	4.1205e+02	3.6702e+02	3.6610e+02
F17	Rank	7	2	6	3	4	5	1
F18	Mean	7.2756e+06	5.6084e+06	4.5584e+06	3.7787e+06	4.1875e+06	4.9041e+06	4.4908e+06
F18	Std	3.8512e+06	5.0590e+06	3.0957e+06	2.7106e+06	4.0514e+06	4.5789e+06	5.9537e+06
F18	Rank	7	6	4	3	2	1	5
F19	Mean	8.0045e+05	4.8009e+05	4.7980e+05	7.6057e+05	6.9719e+05	6.2330e+05	3.9933e+05
F19	Std	6.5671e+05	4.5572e+05	4.6934e+05	6.7229e+05	4.7970e+05	7.6743e+05	3.7500e+05
F19	Rank	7	3	2	6	5	4	1
F20	Mean	1.8825e+03	1.3245e+03	1.2571e+03	1.2365e+03	1.2294e+03	1.2184e+03	1.2271e+03
F20	Std	5.7334e+02	3.3141e+02	3.8054e+02	2.6098e+02	2.5538e+02	3.5825e+02	3.8299e+02
F20	Rank	7	6	5	3	2	1	4
F21	Mean	5.6972e+02	4.4122e+02	4.4437e+02	4.6354e+02	4.5871e+02	4.7897e+02	4.5534e+02
F21	Std	1.1031e+02	4.7606e+01	4.3689e+01	4.9747e+01	5.9135e+01	6.0484e+01	4.4112e+01
F21	Rank	7	1	2	5	4	3	6
F22	Mean	1.3216e+04	6.9619e+03	7.7294e+03	8.4071e+03	7.2869e+03	7.5660e+03	6.6921e+03
F22	Std	4.6770e+03	2.4993e+03	1.9438e+03	1.3152e+03	1.6713e+03	1.6525e+03	2.7835e+03
F22	Rank	7	2	5	6	3	4	1
F23	Mean	8.2787e+02	7.7781e+02	7.7146e+02	7.2023e+02	7.5962e+02	7.7902e+02	7.4559e+02
F23	Std	9.9405e+01	7.3569e+01	7.8397e+01	6.0326e+01	6.5716e+01	9.1469e+01	6.5032e+01
F23	Rank	7	6	2	1	3	4	5
F24	Mean	8.6336e+02	8.2451e+02	8.1922e+02	7.9879e+02	8.0360e+02	8.1433e+02	8.1212e+02
F24	Std	1.0278e+02	9.0172e+01	7.3381e+01	6.4330e+01	7.9883e+01	8.5376e+01	9.5416e+01
F24	Rank	7	5	4	3	2	1	6
F25	Mean	1.0313e+03	7.5854e+02	7.0280e+02	7.3263e+02	7.1641e+02	7.0175e+02	6.7429e+02
F25	Std	1.8958e+02	6.1061e+01	5.3963e+01	6.3682e+01	4.8540e+01	4.8391e+01	4.8784e+01
F25	Rank	7	6	3	5	4	2	1
F26	Mean	4.2407e+03	4.2239e+03	4.5886e+03	3.9304e+03	4.9522e+03	4.9900e+03	3.6759e+03
F26	Std	1.7891e+03	2.1170e+03	2.4640e+03	1.9026e+03	2.0625e+03	2.0538e+03	2.2469e+03
F26	Rank	7	4	5	2	6	3	1
F27	Mean	1.0097e+03	9.3729e+02	9.0871e+02	9.3236e+02	9.3975e+02	9.3139e+02	9.3118e+02
F27	Std	1.0904e+02	9.3439e+01	9.6687e+01	1.0541e+02	1.1859e+02	8.8670e+01	1.2585e+02
F27	Rank	7	5	4	3	2	1	6
F28	Mean	1.1386e+03	8.4466e+02	7.8612e+02	7.9898e+02	8.0476e+02	7.5377e+02	7.1385e+02
F28	Std	2.3783e+02	1.2016e+02	7.7676e+01	7.8621e+01	1.0596e+02	6.6970e+01	7.2659e+01
F28	Rank	7	6	3	4	2	1	5
F29	Mean	2.3898e+03	2.1625e+03	2.2833e+03	2.3400e+03	2.1394e+03	2.2251e+03	2.1508e+03
F29	Std	5.6025e+02	5.2791e+02	4.0905e+02	4.2207e+02	4.5954e+02	4.8974e+02	4.8610e+02
F29	Rank	7	5	4	3	2	1	6
F30	Mean	4.1516e+07	3.5808e+07	3.0986e+07	4.1387e+07	3.6178e+07	3.6134e+07	3.1441e+07
F30	Std	1.2998e+07	1.3729e+07	1.1807e+07	2.2375e+07	1.0602e+07	1.3722e+07	1.2469e+07
F30	Rank	7	3	1	6	5	4	2

APPENDIX E

Tables XXIII–XXVI report the complete condition-wise held-out evaluation results for the six optimizer-assisted FMD methods across the four loads.

TABLE XXIII
DETAILED HELD-OUT EVALUATION RESULTS FOR OPTIMIZER-ASSISTED FMD METHODS AT 0.007 IN AND 0 HP LOAD

Fault condition	Index	IWO-FMD	WO-FMD	PSO-FMD	DE-FMD	HHO-FMD	OOA-FMD
Inner race fault	FCP	0.0982 ±0.0705	0.0878 ±0.0254	0.0899 ±0.0655	0.0972 ±0.0668	0.0692 ±0.0424	0.0666 ±0.0638
Inner race fault	SNR	-6.7663 ±3.1546	-5.6560 ±2.5286	-7.2719 ±3.3503	-6.6712 ±3.5994	-8.5598 ±5.6361	-11.6919 ±7.8411
Inner race fault	CK	3.1848e-04 ± 8.4445e-04	0.0021 ±0.0066	2.0049e-04 ± 6.1389e-04	1.0280e-04 ± 1.8930e-04	8.4902e-05 ± 1.7460e-04	7.0957e-05 ± 1.9993e-04
Outer race fault - 3 o'clock position	FCP	0.1381 ±0.1719	0.1363 ±0.1070	0.0745 ±0.0576	0.1532 ±0.1239	0.1596 ±0.0753	0.1061 ±0.1045
Outer race fault - 3 o'clock position	SNR	-13.2132 ±7.8024	-12.0835 ±4.7871	-18.0929 ±7.9917	-13.1548 ±6.7618	-10.7565 ±4.3205	-16.0520 ±8.4831
Outer race fault - 3 o'clock position	CK	0.0317 ±0.1003	0.0011 ±0.0030	3.7573e-05 ± 1.0219e-04	0.0036 ±0.0088	3.7750e-04 ± 5.9094e-04	5.2529e-04 ± 1.5058e-03
Outer race fault - 6 o'clock position	FCP	0.1049 ±0.0556	0.0920 ±0.0578	0.0887 ±0.0468	0.0759 ±0.0462	0.1125 ±0.0565	0.0675 ±0.0294
Outer race fault - 6 o'clock position	SNR	-13.7627 ±5.2795	-14.8296 ±4.4684	-14.9126 ±4.4434	-16.0224 ±9.6056	-13.1295 ±4.0310	-16.2746 ±5.2621
Outer race fault - 6 o'clock position	CK	0.0011 ±0.0027	9.9443e-04 ± 1.9250e-03	4.2227e-04 ± 7.8302e-04	3.3836e-04 ± 6.2997e-04	5.8586e-04 ± 6.7991e-04	7.0231e-04 ± 2.1817e-03
Outer race fault - 12 o'clock position	FCP	0.0339 ±0.0171	0.0158 ±0.0099	0.0285 ±0.0128	0.0181 ±0.0121	0.0282 ±0.0275	0.0187 ±0.0176
Outer race fault - 12 o'clock position	SNR	-10.4555 ±3.1027	-15.1949 ±4.0200	-9.7891 ±3.3729	-15.7551 ±7.5147	-13.3377 ±5.8599	-16.8068 ±8.3082
Outer race fault - 12 o'clock position	CK	2.0763e-07 ± 4.0120e-07	3.6770e-09 ± 7.6980e-09	2.6326e-07 ± 4.5849e-07	1.8994e-08 ± 4.2532e-08	5.1471e-07 ± 1.5225e-06	5.5050e-07 ± 1.5868e-06
Rolling element fault	FCP	0.0134 ±0.0150	0.0158 ±0.01853	0.0163 ±0.0144	0.0139 ±0.0144	0.0272 ±0.0221	0.0034 ±0.0035
Rolling element fault	SNR	-10.6284 ±7.7985	-8.6283 ±7.3807	-7.0860 ±5.4239	-8.1278 ±7.3363	-4.4905 ±4.0058	-21.3592 ± 10.9949
Rolling element fault	CK	2.7420e-06 ± 6.5666e-06	1.3807e-06 ± 3.4773e-06	2.9783e-06 ± 8.3070e-06	2.7709e-07 ± 4.6235e-07	6.4958e-06 ± 1.8161e-05	3.3915e-10 ± 1.0185e-09

TABLE XXIV
DETAILED HELD-OUT EVALUATION RESULTS FOR OPTIMIZER-ASSISTED FMD METHODS AT 0.007 IN AND 1 HP LOAD

Fault condition	Index	IWO-FMD	WO-FMD	PSO-FMD	DE-FMD	HHO-FMD	OOA-FMD
Inner race fault	FCP	0.1599 ±0.0669	0.1191 ±0.0754	0.1355 ±0.0606	0.1466 ±0.0723	0.1289 ±0.0662	0.1206 ±0.0486
Inner race fault	SNR	-4.4381 ±3.1832	-6.7401 ±3.4424	-5.8956 ±4.3812	-4.9564 ±3.0458	-6.2410 ±3.3411	-5.6101 ±2.2025
Inner race fault	CK	2.0809e-04 ± 2.8178e-04	1.0066e-04 ± 1.6714e-04	1.6352e-04 ± 2.1711e-04	2.5584e-04 ± 3.3437e-04	2.1089e-04 ± 4.7848e-04	1.5119e-04 ± 2.2455e-04
Outer race fault - 3 o'clock position	FCP	0.2052 ±0.2154	0.1939 ±0.2094	0.1377 ±0.2219	0.1161 ±0.1058	0.1778 ±0.1466	0.1250 ±0.1344
Outer race fault - 3 o'clock position	SNR	-14.3207 ±7.1532	-15.2846 ±9.7525	-19.6213 ±8.8303	-17.3005 ±7.8730	-14.2058 ±7.2801	-16.5063 ±6.2056
Outer race fault - 3 o'clock position	CK	0.0094 ±0.0221	0.0188 ±0.0591	0.0172 ±0.0544	5.2810e-04 ± 1.6385e-03	0.0027 ±0.0073	9.1846e-04 ± 2.5930e-03
Outer race fault - 6 o'clock position	FCP	0.2545 ±0.1407	0.1264 ±0.0979	0.2086 ±0.1287	0.1399 ±0.0966	0.1676 ±0.1120	0.0901 ±0.0357
Outer race fault - 6 o'clock position	SNR	-8.3826 ±4.5306	-14.6390 ±7.9036	-8.2329 ±2.5911	-13.1376 ±5.6973	-12.6609 ±7.9340	-14.5817 ±3.6088
Outer race fault - 6 o'clock position	CK	0.0149 ±0.0262	0.0022 ±0.0070	0.0134 ±0.0257	0.0021 ±0.0059	0.0019 ±0.0036	8.8795e-05 ± 9.7326e-05
Outer race fault - 12 o'clock position	FCP	0.0306 ±0.0182	0.0322 ±0.0207	0.0444 ±0.0254	0.0406 ±0.0292	0.0385 ±0.0194	0.0340 ±0.0177
Outer race fault - 12 o'clock position	SNR	-10.9275 ±6.1451	-10.8399 ±4.7729	-8.0434 ±4.2976	-9.5983 ±7.4669	-9.4000 ±4.8377	-9.0061 ±3.4206
Outer race fault - 12 o'clock position	CK	2.9314e-07 ± 5.2386e-07	1.8578e-07 ± 3.2700e-07	8.8977e-06 ± 2.4976e-05	1.9517e-06 ± 4.3375e-06	3.6706e-07 ± 4.4892e-07	7.8972e-07 ± 2.0959e-06
Rolling element fault	FCP	0.0155 ±0.0121	0.0114 ±0.0099	0.0138 ±0.0099	0.0117 ±0.0073	0.0113 ±0.0045	0.0086 ±0.0091
Rolling element fault	SNR	-6.6165 ±4.1373	-9.2558 ±5.8553	-9.0587 ±5.9251	-7.4638 ±4.3069	-6.9060 ±4.0498	-12.2489 ±8.7931
Rolling element fault	CK	3.5880e-06 ± 1.0968e-05	4.0648e-07 ± 6.7629e-07	8.8786e-08 ± 1.9192e-07	1.8145e-07 ± 3.7230e-07	1.0653e-06 ± 2.4804e-06	6.5799e-07 ± 1.5510e-06

TABLE XXV
DETAILED HELD-OUT EVALUATION RESULTS FOR OPTIMIZER-ASSISTED FMD METHODS AT 0.007 IN AND 2 HP LOAD

Fault condition	Index	IWO-FMD	WO-FMD	PSO-FMD	DE-FMD	HHO-FMD	OOA-FMD
Inner race fault	FCP	0.1240 $\pm$ 0.0373	0.0974 $\pm$ 0.0470	0.1247 $\pm$ 0.0353	0.1072 $\pm$ 0.0424	0.1089 $\pm$ 0.0630	0.1193 $\pm$ 0.0535
Inner race fault	SNR	-6.1590 $\pm$ 3.3681	-8.2212 $\pm$ 4.5294	-6.7568 $\pm$ 1.8616	-7.5823 $\pm$ 3.2266	-8.6074 $\pm$ 4.5811	-7.2578 $\pm$ 3.4782
Inner race fault	CK	3.6842e-04 $\pm$ 8.8951e-04	8.9939e-05 $\pm$ 1.2902e-04	1.2543e-04 $\pm$ 1.7391e-04	2.2260e-04 $\pm$ 4.7926e-04	1.6697e-04 $\pm$ 3.0540e-04	2.1170e-04 $\pm$ 3.5889e-04
Outer race fault - 3 o'clock position	FCP	0.1457 $\pm$ 0.1431	0.2152 $\pm$ 0.1913	0.1821 $\pm$ 0.2453	0.2183 $\pm$ 0.2543	0.2013 $\pm$ 0.1274	0.1314 $\pm$ 0.1536
Outer race fault - 3 o'clock position	SNR	-16.4146 $\pm$ 8.6147	-15.0297 $\pm$ 9.5876	-14.3371 $\pm$ 5.2584	-15.5276 $\pm$ 9.2147	-12.7422 $\pm$ 6.6741	-17.7736 $\pm$ 7.1065
Outer race fault - 3 o'clock position	CK	0.0033 $\pm$ 0.0072	0.0111 $\pm$ 0.0226	0.0457 $\pm$ 0.1444	0.0524 $\pm$ 0.1549	0.0026 $\pm$ 0.0052	0.0011 $\pm$ 0.0022
Outer race fault - 6 o'clock position	FCP	0.1739 $\pm$ 0.1119	0.1401 $\pm$ 0.0568	0.0973 $\pm$ 0.0577	0.1381 $\pm$ 0.0515	0.1723 $\pm$ 0.0822	0.1074 $\pm$ 0.0715
Outer race fault - 6 o'clock position	SNR	-11.2755 $\pm$ 5.6075	-11.3852 $\pm$ 2.2287	-13.9559 $\pm$ 3.3816	-11.0024 $\pm$ 3.1332	-10.5832 $\pm$ 5.5201	-14.2799 $\pm$ 5.8014
Outer race fault - 6 o'clock position	CK	0.0011 $\pm$ 0.0018	2.5133e-04 $\pm$ 6.7388e-04	3.1972e-05 $\pm$ 8.9919e-05	9.0263e-05 $\pm$ 1.5449e-04	7.6601e-04 $\pm$ 1.7980e-03	3.4813e-05 $\pm$ 4.9353e-05
Outer race fault - 12 o'clock position	FCP	0.0543 $\pm$ 0.0453	0.0731 $\pm$ 0.0717	0.0844 $\pm$ 0.0643	0.0493 $\pm$ 0.0259	0.0656 $\pm$ 0.0405	0.0346 $\pm$ 0.0298
Outer race fault - 12 o'clock position	SNR	-13.7473 $\pm$ 9.5227	-9.5052 $\pm$ 5.9671	-8.7468 $\pm$ 5.7801	-10.2088 $\pm$ 4.4774	-9.1523 $\pm$ 5.2649	-13.8034 $\pm$ 6.4435
Outer race fault - 12 o'clock position	CK	4.9164e-06 $\pm$ 1.4340e-05	4.7364e-06 $\pm$ 1.0086e-05	2.1181e-06 $\pm$ 3.5968e-06	1.9482e-07 $\pm$ 3.0277e-07	1.2295e-06 $\pm$ 3.2676e-06	6.3738e-08 $\pm$ 1.5760e-07
Rolling element fault	FCP	0.0204 $\pm$ 0.0136	0.0203 $\pm$ 0.0186	0.0138 $\pm$ 0.0149	0.0131 $\pm$ 0.0073	0.0093 $\pm$ 0.0057	0.0096 $\pm$ 0.0075
Rolling element fault	SNR	-6.9435 $\pm$ 6.7252	-7.5780 $\pm$ 8.0028	-9.5786 $\pm$ 5.7793	-7.6645 $\pm$ 6.5092	-9.3633 $\pm$ 6.6249	-10.3077 $\pm$ 6.6864
Rolling element fault	CK	1.2142e-05 $\pm$ 3.2395e-05	7.7600e-07 $\pm$ 1.4072e-06	4.1811e-07 $\pm$ 9.4389e-07	7.1338e-07 $\pm$ 1.6708e-06	6.9007e-08 $\pm$ 1.0957e-07	1.0812e-06 $\pm$ 3.0017e-06

TABLE XXVI
DETAILED HELD-OUT EVALUATION RESULTS FOR OPTIMIZER-ASSISTED FMD METHODS AT 0.007 IN AND 3 HP LOAD

Fault condition	Index	IWO-FMD	WO-FMD	PSO-FMD	DE-FMD	HHO-FMD	OOA-FMD
Inner race fault	FCP	0.0758 $\pm$ 0.0450	0.0511 $\pm$ 0.0238	0.0773 $\pm$ 0.0238	0.0656 $\pm$ 0.0394	0.0548 $\pm$ 0.0192	0.0620 $\pm$ 0.0322
Inner race fault	SNR	-8.8014 $\pm$ 5.9002	-11.2443 $\pm$ 4.3367	-7.4260 $\pm$ 2.5449	-10.1644 $\pm$ 5.0805	-10.5223 $\pm$ 3.6990	-10.1654 $\pm$ 4.8714
Inner race fault	CK	9.2444e-05 $\pm$ 2.1001e-04	1.6531e-06 $\pm$ 1.7304e-06	3.2012e-05 $\pm$ 7.9414e-05	1.2452e-05 $\pm$ 2.1445e-05	2.6887e-06 $\pm$ 2.8291e-06	3.0370e-05 $\pm$ 8.0914e-05
Outer race fault - 3 o'clock position	FCP	0.0840 $\pm$ 0.1159	0.0443 $\pm$ 0.0450	0.1358 $\pm$ 0.1713	0.0628 $\pm$ 0.0488	0.2187 $\pm$ 0.1546	0.0351 $\pm$ 0.0298
Outer race fault - 3 o'clock position	SNR	-21.3322 $\pm$ 9.9561	-22.9995 $\pm$ 9.3745	-16.8491 $\pm$ 9.7192	-19.5148 $\pm$ 6.0045	-10.1446 $\pm$ 5.9668	-24.5602 $\pm$ 9.5456
Outer race fault - 3 o'clock position	CK	0.0014 $\pm$ 0.0045	3.6957e-06 $\pm$ 8.6311e-06	0.0040 $\pm$ 0.0085	3.0703e-06 $\pm$ 6.9140e-06	0.0033 $\pm$ 0.0088	2.4233e-05 $\pm$ 7.5755e-05
Outer race fault - 6 o'clock position	FCP	0.2678 $\pm$ 0.1630	0.2008 $\pm$ 0.1346	0.1775 $\pm$ 0.0989	0.1783 $\pm$ 0.1227	0.1993 $\pm$ 0.0854	0.1390 $\pm$ 0.0898
Outer race fault - 6 o'clock position	SNR	-7.1196 $\pm$ 4.4465	-8.4852 $\pm$ 4.7382	-10.6133 $\pm$ 7.3884	-9.4749 $\pm$ 3.4621	-7.5099 $\pm$ 2.0912	-10.6058 $\pm$ 4.3147
Outer race fault - 6 o'clock position	CK	0.0049 $\pm$ 0.0089	0.0023 $\pm$ 0.0061	1.9148e-04 $\pm$ 2.8278e-04	0.0021 $\pm$ 0.0060	5.2366e-04 $\pm$ 1.0415e-03	9.8698e-05 $\pm$ 1.4303e-04
Outer race fault - 12 o'clock position	FCP	0.0332 $\pm$ 0.0331	0.0366 $\pm$ 0.0302	0.0322 $\pm$ 0.0170	0.0287 $\pm$ 0.0203	0.0323 $\pm$ 0.0160	0.0282 $\pm$ 0.0296
Outer race fault - 12 o'clock position	SNR	-13.6051 $\pm$ 7.3036	-12.5510 $\pm$ 6.1135	-12.2851 $\pm$ 5.4502	-12.4318 $\pm$ 6.9080	-11.2693 $\pm$ 4.2159	-14.8910 $\pm$ 7.0847
Outer race fault - 12 o'clock position	CK	1.8133e-07 $\pm$ 4.8212e-07	2.1163e-07 $\pm$ 6.0280e-07	5.5971e-08 $\pm$ 1.1005e-07	1.9737e-06 $\pm$ 6.0801e-06	3.7554e-08 $\pm$ 5.8247e-08	1.1945e-07 $\pm$ 3.6770e-07
Rolling element fault	FCP	0.0129 $\pm$ 0.0099	0.0130 $\pm$ 0.012	0.0205 $\pm$ 0.0185	0.0143 $\pm$ 0.0089	0.0210 $\pm$ 0.0147	0.0142 $\pm$ 0.0102
Rolling element fault	SNR	-9.5807 $\pm$ 4.1816	-10.8980 $\pm$ 6.3191	-7.3827 $\pm$ 5.1121	-9.3327 $\pm$ 6.1065	-6.0807 $\pm$ 3.3660	-8.4052 $\pm$ 7.3880
Rolling element fault	CK	3.6613e-08 $\pm$ 7.2592e-08	8.2393e-08 $\pm$ 2.4197e-07	4.4411e-06 $\pm$ 1.3537e-05	3.8056e-06 $\pm$ 1.1355e-05	8.6075e-07 $\pm$ 1.9132e-06	4.6162e-07 $\pm$ 1.2663e-06

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